

This is the first of 30 revision sessions.

Each session contains a number of questions for you to try.

If you work through them all, you should have a good chance of gaining a Grade 4 in your GCSE.

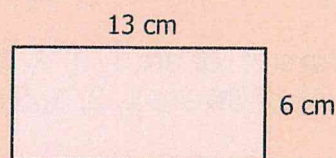
Answers to the questions are provided. Use these to check your work. If you get a question wrong, see if you can work out where you went wrong.

There are reminders to help you. These are always on the left-hand side. Your teacher will also be pleased to help if you are stuck.

Quick Quiz

- (1) Work out 4×278 without a calculator.
- (2) Write down a two-digit factor of 20, other than 20 itself.
- (3) Work out the value of $2(a + b)$ when $a = 6$ and $b = 10$
- (4) How many metres are there in a kilometre?

- (5) What is the perimeter of this rectangle?



- (6) Draw all the lines of symmetry on the rectangle.

The focus for today's session is...

Prime numbers, HCFs and LCMs

Reminder

A prime number has **no factors** except 1 and itself.

- **7** is a prime – its only factors are 1 and 7.
- 8 has other factors (2 and 4), so it is not a prime.

1 is **not** a prime number. (This is because prime numbers have exactly two factors, and 1 only has one.)

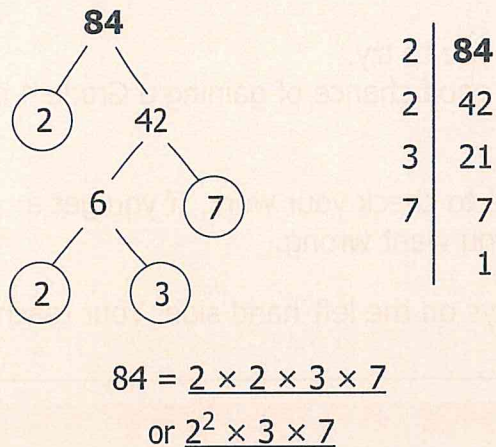
Practice Questions

- (1) Which of these are prime numbers?

5	9	13	18
19	25	33	42
- (2) Write down the first ten prime numbers.
- (3) Which is the largest prime number less than 50?

Reminder

Here are two different ways to express 84 as a product of prime factors.



Practice Questions

- (4) Write these integers as the product of prime factors

- (a) 66
- (b) 40
- (c) 64
- (d) 54
- (e) 135
- (f) 250
- (g) 106
- (h) 1,000

Reminder

The **factors** of **20** are 1, 2, 4, 5, 10 and 20.
The factors of **30** are 1, 2, 3, 5, 6, 10 and 30.

The **common factors** are 1, 2, 5 and 10
The **highest** common factor (**HCF**) is 10.

Multiples of **20** are 20, 40, 60, 100, 120, etc ...
Multiples of **30** are 30, 60, 90, 120, 150, etc ...

Common multiples are 60, 120, etc ...
The **lowest** common multiple (**LCM**) is 60.

Practice Questions

- (5) Work out the HCF of:

- (a) 12 and 16
- (b) 15 and 30
- (c) 18 and 35
- (d) 54 and 90

- (6) Work out the LCM of:

- (a) 6 and 8
- (b) 5 and 11
- (c) 12 and 20
- (d) 36 and 54

Reasoning and Problem-Solving

- (7) Debbie says:

When you add two prime numbers, you will always get an even number.

Is Debbie correct? Explain your answer.

- (8) The number $P = 2^4 \times 5^2 \times 11$. The number $Q = 2^2 \times 5^3 \times 7$.

Express $P \times Q$ as the product of prime factors.

- (9) Phil can see two lighthouses, A and B.

The light from Lighthouse A flashes every 24 seconds.

The light from Lighthouse B flashes every 20 seconds.

Phil sees A and B flash at the same time.

How long will it be before this next happens?

Quick Quiz

- (1) The temperature in London is 3°C . The temperature in Moscow is -15°C . How much colder is it in Moscow than in London?
- (2) Write these decimals in order, smallest first:
0.021 0.21 0.012 0.12
- (3) What is the order of rotation symmetry of a rectangle?
- (4) What is the value of 4^3 ?

Review of Session 1

- (1) Write down two prime numbers which add up to 30.
- (2) Express 120 as the product of prime factors.
- (3) Work out the lowest common multiple of 60 and 48.
- (4) What is the highest common factor of 42 and 70?
- (5) Write down three common factors of 36 and 60.

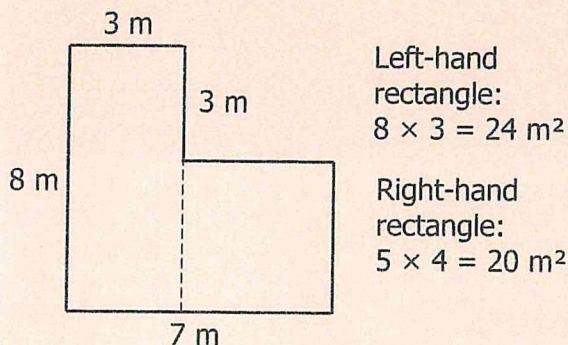
The focus for today's session is...

Area

Reminder

To find the area of a rectangle, multiply the length by the width.

The area of a shape like the one below can be worked out by splitting it into rectangles.



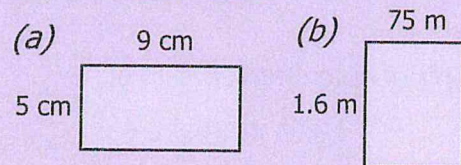
Now, add the two areas together:

$$\text{Total} = 24 + 20 = \underline{44 \text{ m}^2}$$

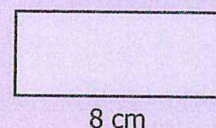
(Remember that units of area are squared units: m^2 , cm^2 etc.)

Practice Questions

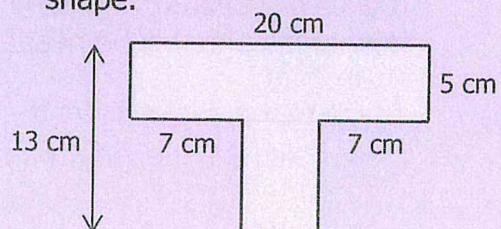
- (1) Work out the area of these rectangles, giving the correct units in each case:



- (2) The area of this rectangle is 36 cm^2 . What is its height?

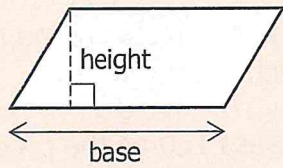


- (3) Work out the area of this T-shape.

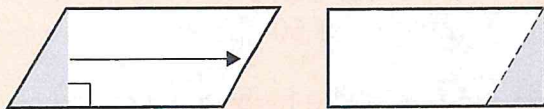


Reminder

To work out the area of a parallelogram, multiply the base by the perpendicular height.

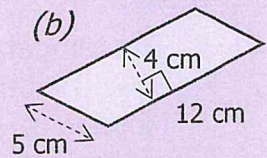
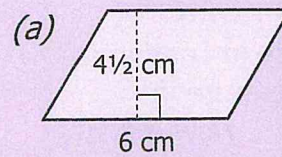


The rule is the same as for a rectangle.

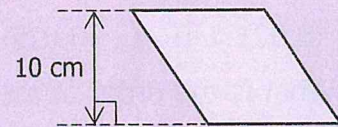


Practice Questions

- (4) Work out the areas of these parallelograms.

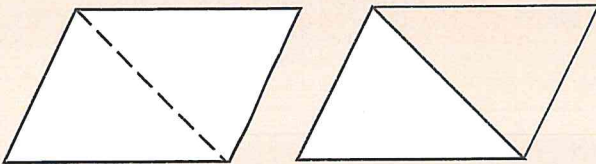


- (5) The area of this parallelogram is 68 cm^2 . What is the length of its base?



Reminder

A triangle is half a parallelogram,



so the area of a triangle can be worked out like this:

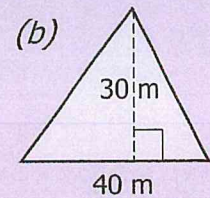
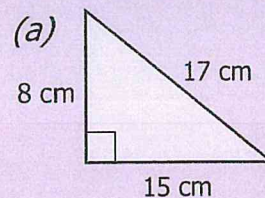
$$\frac{\text{base} \times \text{perpendicular height}}{2}$$

Many people shorten this to:

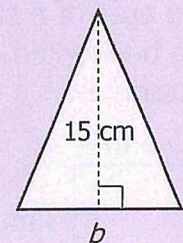
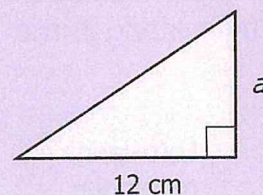
$$\text{base} \times \text{height} \div 2$$

Practice Questions

- (6) Work out the areas of these triangles.



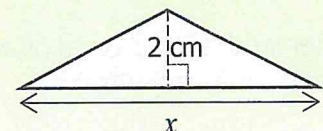
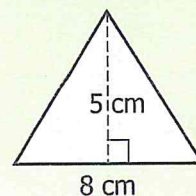
- (7) These two triangles both have area 30 cm^2 . Work out the lengths a and b .



Reasoning and Problem-Solving

- (8) A rectangular floor measures 8 metres by 5 metres. It is to be covered with square tiles. Each tile measures 50 cm by 50 cm. How many tiles will be needed?

- (9) These two triangles have the same area. Calculate the length marked x .

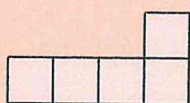


Quick Quiz

- (1) Work out $834 \div 6$, without a calculator.
- (2) Work out $11^2 - 7^2$, without a calculator.
- (3) An accurate plan of a hall is drawn, using a scale of 1 : 200.

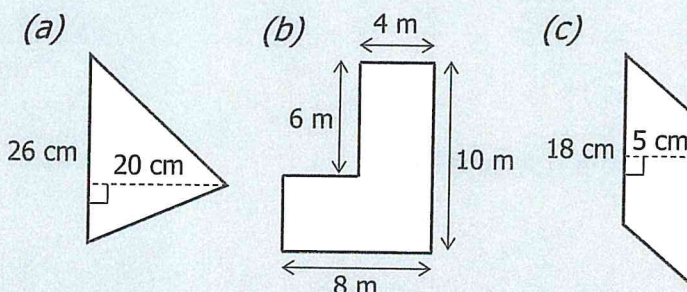
A ramp is 3 cm long on the plan. What is the real length of the ramp, in metres?

- (4) Add a square so that this is a net of a cube.

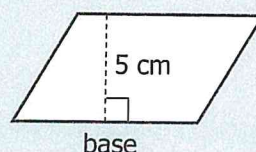


Review of Session 2

- (1) Work out the areas of these shapes.



- (2) The area of this parallelogram is 42 cm^2 . Work out the length of the base.



The focus for today's session is...

Fractions and ratios

Reminder

To simplify a fraction, **divide** the numerator and denominator by the **same number**:

$$\frac{4}{10} = \frac{2}{5} \quad \frac{24}{54} = \frac{4}{9}$$

You can simplify a ratio in the same way:

Simplify 8 : 28 by dividing both numbers by 4:

$$8 : 28 = 2 : 7$$

Practice Questions

- (1) Simplify each fraction as far as possible:
(a) $\frac{8}{18}$ (b) $\frac{15}{21}$ (c) $\frac{35}{100}$ (d) $\frac{30}{45}$
- (2) Simplify each ratio as far as possible:
(a) 6 : 14 (b) 25 : 35
(c) £1.20 : 80p (d) 2.4 m : 66 cm
- (3) In Year 11 there are 80 girls and 64 boys. What is the ratio of girls : boys, in its simplest form?
- (4) I go shopping with £60 and when I return I have £25 left. What fraction of the money did I spend? Give the answer in its simplest form.

Reminder

If you **multiply** the numerator and denominator of a fraction by the **same number**, you get an equivalent fraction.

$$\frac{2}{3} \xrightarrow{\times 5} \frac{10}{15} \quad \frac{3}{7} \xrightarrow{\times 9} \frac{27}{63}$$

The same rule also works for ratios.

$$4 : 5 \xrightarrow{\times 2} 8 : 10$$
$$4 : 5 \xrightarrow{\times 3} 12 : 15$$
$$4 : 5 \xrightarrow{\times 10} 40 : 50$$

Practice Questions

- (5) Find the missing number in each of these:

$$(a) \frac{5}{6} = \frac{\square}{18} \quad (b) \frac{7}{10} = \frac{77}{\square}$$

- (6) Find the missing number in each of these:

$$(a) 3 : 4 = \square : 16$$

$$(b) 6 : 11 = 48 : \square$$

- (7) Which of these fractions is the larger?

$$\frac{3}{5} \text{ or } \frac{5}{8}$$

Reminder

Be careful when converting between fractions and ratios.

Example: The ratio of blue sweets to red sweets in a bag is 2 : 3. What fraction of the sweets are blue?

Solution: 2 represents blue
3 represents red
5 represents the total

(B) (B)

(R) (R) (R)

So the fraction is $\frac{2}{5}$.

Practice Questions

- (8) Andie and Bob share some money in the ratio 4 : 7. What fraction of the money does Andie get?

- (9) $\frac{1}{4}$ of a class are girls.
What is the ratio girls : boys?

- (10) Harry is late for school $\frac{2}{7}$ of the time.

What is the ratio of days he is late to the days he is on time?

Reasoning and Problem-Solving

- (11) In a car park, the ratio of cars to vans is 5 : 1.
The ratio of vans to motorbikes is 2 : 3.

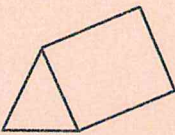
What is the ratio of cars to motorbikes?

- (12) Write these three fractions in order, starting with the smallest.

$$\frac{3}{10} \quad \frac{1}{3} \quad \frac{4}{15}$$

You must show your working.

Quick Quiz

- (1) How many faces are there on a triangular prism? 
- (2) Put these types of angles in increasing order:

obtuse	right-angle
reflex	acute
- (3) Work out 125×7 , without a calculator.
- (4) Which of these is not a square number?

2	4	25	100
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- (5) Work out the value of $\frac{n}{4} - 10$ when $n = 60$.

Review of Session 3

- (1) Write the ratio 400 g : 1.4 kg in its simplest form
- (2) Write the fraction $\frac{24}{60}$ in its simplest form.
- (3) Which of these is the larger?
 $\frac{3}{7}$ or $\frac{4}{9}$
 Show how you got your answer.
- (4) $\frac{5}{8}$ of the membership of a club is aged under 18. What is the ratio under 18 : 18 or over?
- (5) Work out the missing number:
 $5 : 12 = \square : 108$

The focus for today's session is... Probability

Reminder

A probability can be expressed as:

- a fraction
- a decimal; or
- a percentage

Never express it as a ratio.

To work out the probability of an event happening, subtract from 1.

Example

If the probability of winning a raffle is 0.12, the probability of not winning is:

$$1 - 0.12 = \underline{0.88}$$

Practice Questions

- (1) Express the following probabilities as fractions:
 - (a) rolling a 3 on a fair dice
 - (b) choosing a vowel if a letter of the alphabet is chosen at random
 - (c) a person chosen at random has their birthday in April.
- (2) The probability it will rain on Friday is 0.34. What is the probability it will not rain?
- (3) The probability of being selected for a team is $\frac{6}{17}$. What is the probability of not being selected?

Reminder

To work out a probability, you need to know the number of possible outcomes.

Example: A fair coin is spun twice. What is the probability of getting two heads?

Solution: The possible outcomes are

HH, HT, TH, TT.

One of these is 'two heads', so the probability is $\frac{1}{4}$.

Practice Questions

- (4) Four names, Abbie, Bob, Charlotte and Diana, are placed in a bag, and two are selected at random.
- (a) List all the possible pairs that could be selected.
 - (b) What is the probability that Abbie is chosen, but not Bob?
- (5) (a) List the possible outcomes when a fair coin is spun three times.
- (b) What is the probability of getting Heads exactly twice?

Reminder

Often, it is useful to set out the equally likely outcomes in a table.

For example, when two fair dice are thrown and the scores added, the table would look like this:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(Use this table to help answer Q6.)

Practice Questions

- (6) When two fair dice are thrown, work out the probability that:
- (a) the total is 9 or more;
 - (b) the scores on the two dice are the same.
- (7) A fair dice is rolled and a fair coin is spun.
- (a) List all the equally likely outcomes.
(Use a table if you prefer.)
 - (b) What is the probability of getting Heads and a number greater than 2?

Reasoning and Problem-Solving

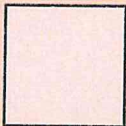
- (8) Alexa says: *There are only three outcomes to a football match – Win, Lose or Draw. So the chances of winning must always be one-third.*

Explain why Alexa is wrong.

- (9) A bag contains only white counters and black counters. The total number of counters is less than 10.

When a counter is drawn at random, the probability of drawing a white counter is 0.2. How many black counters are there in the bag?

Quick Quiz

- (1) Boxes of chocolates cost £*c* each. What is the cost of 8 boxes of chocolates, in £?
- (2) Which of these is not a multiple of 12?
60 12 6
- (3) Draw all the lines of symmetry on this square. 
- (4) How many ml are there in 1 l?
- (5) Draw a triangle with an obtuse angle.

Review of Session 4

- (1) Jane has the following coins in her pocket:
 $2 \times \text{£1 coins}$
 $1 \times 50 \text{ pence piece}$
 $3 \times 20 \text{ pence pieces}$
 $2 \times 10 \text{ pence pieces}$
 She draw a coin out of her pocket at random. What is the probability it is worth 50 p or more?
- (2) The probability of winning a prize in a raffle is 0.06. What is the probability of not winning a prize?
- (3) The numbers from 1 to 5 are placed in a hat. Two of the numbers are pulled out at random. What is the probability the sum of the two digits is 6 or more?

The focus for today's session is... Negative numbers

Reminder

When adding or subtracting negative numbers, draw a number line.

Examples

To do $3 - 7$, start at 3 and go down 7.

Answer = **-4**

To do $(-4) + 1$, start at -4 and go up 1.

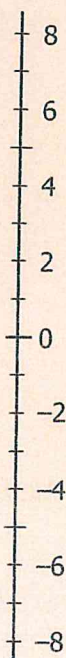
Answer = **-3**

$+-$ can be replaced by $-$

- $6 + (-2) = 6 - 2 = 4$
- $(-7) + (-2) = (-7) - 2 = -9$

$--$ can be replaced by $+$

- $3 - (-2) = 3 + 2 = 5$
- $(-8) - (-10) = (-8) + 10 = 2$



Practice Questions

- (1) Work out the following.
(Don't rush – take care to get them right.)
 - (a) $(-2) + 5$
 - (b) $(-6) - 1$
 - (c) $3 + (-2)$
 - (d) $5 - (-3)$
 - (e) $(-1) + (-4)$
 - (f) $1 - 7$
 - (g) $(-3) - (-1)$
 - (h) $7 + (-4)$
 - (i) $(-6) + (-3)$
 - (j) $(-6) - (-8)$

Reminder

To multiply or divide, follow some simple rules.

$$\text{pos} \times \text{neg} = \text{neg}$$

- $5 \times (-3) = -15$
- $(-4) \times 6 = -24$

$$\text{neg} \times \text{neg} = \text{pos}$$

- $(-8) \times (-2) = 16$
- $(-5)^2 = (-5) \times (-5) = 25$

$$\text{same rules for division}$$

- $30 \div (-10) = -3$
- $(-21) \div (-3) = 7$

Practice Questions

(2) Work out the following:

- (a) $6 \times (-2)$ (b) $(-15) \div (-5)$
(c) $(-20) \div 4$ (d) $(-4) \times (-3)$
(e) $(-9) \times 1$ (f) $36 \div (-6)$
(g) $(-7)^2$ (h) $(-8) \div (-1)$
(i) $56 \div (-7)$ (j) $13 \times (-2)$

(3) Here is a mixture:

- (a) $(-3) - (-7)$ (b) $(-6) \times (-5)$
(c) $(-5) - 3$ (d) $28 \div (-4)$
(e) $(-9) + (-2)$ (f) $(-63) \div (-9)$

Reminder

The same rules apply when simplifying in algebra.

Examples

- $(-6m) + 4m = -2m$
- $(-a) - 4a = -5a$
- $(-2) \times 4y = -8y$

For longer expressions, remember to consider the **sign** before each term.

Example

$$2a - 3b - 5a - 2b + 7a$$

Group the 'a' terms like this:

$$(2a) - 3b (-5a) - 2b (+7a)$$

$$2a - 5a + 7a = 4a$$

$$\text{Likewise, } -3b - 2b = -5b$$

$$\text{So the expression} = 4a - 5b$$

Practice Questions

(4) Simplify the following:

- (a) $(-n) + 3n$
(b) $-5 \times (-2p)$
(c) $(-2x) - 6x$
(d) $(-4t) \times 3$
(e) $2y \times (-6y)$
(f) $(-h) + (-3h) - 5h$

(5) Simplify the following.
(Take time over each one.)

- (a) $3a - b - 2a + 4b$
(b) $(-6p) - q - 2p + 4q$
(c) $6x - 3 - x - 7 - 3x$
(d) $(-2s) + (-2s) + (-4t) + s$
(e) $3x - 7y - 2x - y + 5x$
(f) $9m - 4 + (-12m) - 6 + 4m$

Reasoning and Problem-Solving

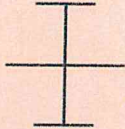
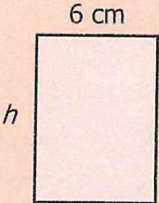
(6) Three of the four expressions in the box add up to n . Which three?

(7) Eloise says: *Two minuses make a plus.*
So $(-5) + (-2)$ must equal 7.

Explain why Eloise is wrong.

- A** $m + 3n + p$
B $(-3m) + (-4n) + 2p$
C $(-2m) - 3n - 3p$
D $2m + 2n + (-3p)$

Quick Quiz

- (1) What is the order of rotation symmetry of this logo? 
- (2) Work out $476 \div 7$ without a calculator.
- (3)  The perimeter of this rectangle is 30 cm. Work out the length marked h .
- (4) What is the sixth square number?

Review of Session 5

- (1) Work out the following, without a calculator:
 - (a) $(-1) + (-4)$
 - (b) $3 \times (-5)$
 - (c) $7 - 10$
 - (d) $(-24) \div 6$
 - (e) $(-5) \times (-7)$
 - (f) $(-8) - (-2)$
 - (g) $(-6) \div (-1)$
 - (h) $(-5) - 3$
- (2) Simplify these expressions:
 - (a) $4a - 7a$
 - (b) $-3 \times (5p)$
 - (c) $2x - 3y - 6x - y$
 - (d) $-2n + 6 + (-n) - 9 + 5n$

The focus for today's session is... Direct Proportion

Reminder

Example

Lennie works as a waiter in a restaurant. This weekend, he is paid £66 for 8 hours' work.

Next weekend he will work for 12 hours. How much will he be paid?

Solution

There are several ways to solve this type of question. Some people make a simple table, like this:

	Hours	Pay (£)	
$\div 8$	8	66	$\div 8$
$\times 12$	1	8.25	$\times 12$
	12	99	

Answer: £99

Practice Questions

- (1) A bricklayer lays 228 bricks in 4 hours. How many bricks will she lay in 10 hours?
- (2) Jonty buys 6 chocolate bars for £2.94. What would be the cost of 8 identical chocolate bars?
- (3) Celia is typing a document. It takes her 15 minutes to type the first 930 words.
It takes her a total of 35 minutes to type the complete document. How many words does the document contain?
- (4) At a supermarket, 600 grams of tomatoes cost £1.26. What would 1.1 kg of tomatoes cost from the same supermarket?

Reminder

A similar method can be used for scaling recipes.

Example

A recipe for 4 pancakes uses 130 ml of milk.

How much milk will be needed to make 18 pancakes?

Solution

	Pancakes	Milk	
$\div 2$	4	130	$\div 2$
$\times 9$	2	65	$\times 9$
	18	585	

Answer: 585 ml

Practice Questions

(5) To make 24 cupcakes, you need:

- 250 g sugar
- 4 tablespoons milk

(a) How much sugar would be needed to make 18 cupcakes?

(b) How much milk would be needed for 36 cupcakes?

(6) To make vegetable lasagne for 6 people, you need 125 g cheese.

You actually have 300 g cheese.

For how many people can you make a vegetable lasagne?

(Assume you have enough of everything else.)

Reminder

Value for Money questions can often be answered by working out the cost per unit, or the amount per £.

Example

At a supermarket, a 2.5 kg bag of potatoes costs £2.00. A 1.75 kg bag costs £1.50. Which is best value for money?

Solution

Work out the cost per kg for each:

$$2.00 \div 2.5 = £0.80$$

$$1.50 \div 1.75 = £0.86$$

The 2.5 kg bag costs less per kg, so it is better value.

Practice Questions

(7) Roz is buying toilet roll in bulk.

At the local supermarket, she could buy 36 rolls for £15.12.

Buying online, 54 rolls of the same brand would cost £23.60.

Which of these is better value for money? Show your working.

(8) At Shop A, a 450 g box of breakfast cereal costs £1.80.

At Shop B, a 650 g box of the same cereal costs £2.75.

Which of these is the best value for money? You must show your working.

Reasoning and Problem-Solving

(9) To buy 50 packets of crisps, which of these offers works out cheapest? Show all your working.

A: Packets of 6 for £3.20, individual packets 75 p each.

B: Individual packets 80 p each; buy two packets get one free

Quick Quiz

- (1) What is the value of $\sqrt{144}$?
- (2) Prize money of £500 is shared equally between P people. Write an expression for the amount of money each person received.
- (3) How many vertices are there on a cube?
- (4) At the top of the Answer Sheet there is a triangle. Measure the size of the smallest angle.
- (5) What is the highest common factor of 12 and 28?

Review of Session 6

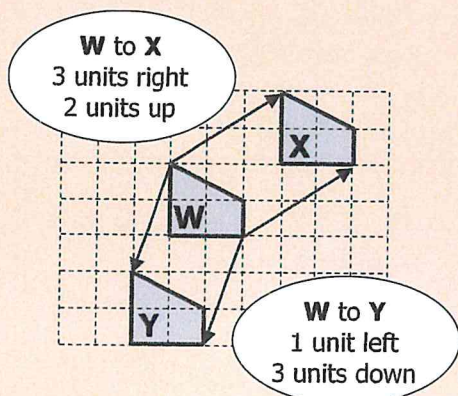
- (1) 6 apples cost £2.04. What is the price of 10 apples?
- (2) At a supermarket, 350 g of Cheddar costs £4.20. How much will 500 g cost?
- (3) To make pizza for 4 people, you need 550 g of flour. How much flour would you need if you were making pizza for 14 people?
- (4) Asa bought 15 litres of petrol for £19.05. Noah bought 22 litres for £28.60. Who got better value for money? Explain your answer.

The focus for today's session is...

Translations, reflections and rotations

Reminder

When you move a shape, without rotating or reflecting it, this is known as a **translation**.



Vectors are often used to describe translations.

W to X	W to Y
3 units right	1 unit left
2 units up	3 units down
$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	$\begin{pmatrix} -1 \\ -3 \end{pmatrix}$

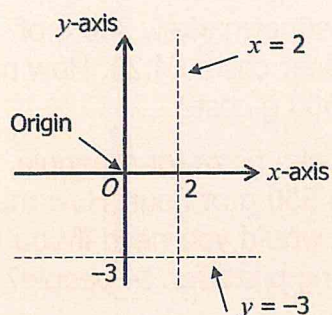
Practice Questions

- (1) On **Diagram 1** on the Answer Sheet:
 - (a) Translate Shape **A** by 3 units to the right and 1 unit up.
 - (b) Translate Shape **B** by $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$
 - (c) Translate Shape **C** by $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$
- (2) Look at **Diagram 2** on the Answer Sheet. Use vectors to describe the following translations:
 - (a) **P to Q**
 - (b) **R to S**
 - (c) **T to U**

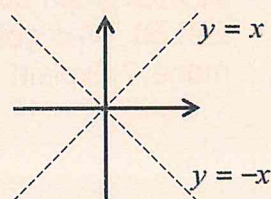
Reminder

To draw a **reflection**, you need to know where the mirror line is.

Here is a reminder about horizontal and vertical lines.



You also need to know the equations of the two diagonal lines.



Practice Questions

- (3) On **Diagram 3**, write down the equations of the lines (a) to (d).
- (4) On **Diagram 4**, draw these straight lines:
(a) $x = -3$ (b) $y = 4$
(c) $y = x$
- (5) On **Diagram 5**:
(a) Reflect **A** in the x -axis.
(b) Reflect **B** in $x = -1$.
(c) Reflect **C** in $y = -x$.
- (6) Look at **Diagram 6**:
What is the mirror line for each of these reflections?
(a) **P** to **Q** (b) **R** to **S**
(c) **T** to **U**

Reminder

To describe a **rotation**, you need:

- the centre of rotation
- the angle of rotation
- clockwise or anti-clockwise

You may find rotations easier if you use tracing paper. (Plain white paper may be OK.)

To find the centre of rotation, try different possibilities until you find the right one. (It may take a few tries.)

Practice Questions

- (7) On **Diagram 7**:
(a) Rotate **A** about the origin, through 90° clockwise.
(b) Rotate **B** about $(0, 3)$ through 180° .
(c) Rotate **C** about $(-1, 1)$ through 90° anti-clockwise.
- (8) Look at **Diagram 8**:
Describe each of these rotations:
(a) **A** onto **B** (b) **C** onto **D**
(c) **E** onto **F**

Reasoning and Problem-Solving

- (9) Look at **Diagram 9**. There are **five** different ways to map square **P** onto square **Q**. How many can you find? Describe each one fully.
- (10) Look at **Diagram 10**. Explain clearly why Shape **X** cannot be mapped to Shape **Y** using a translation, reflection or rotation.

ANSWER SHEET FOR SESSION 7

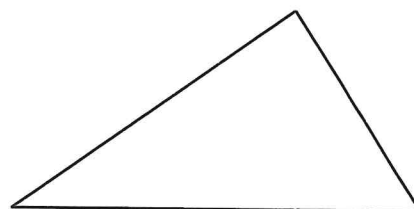


Diagram 1

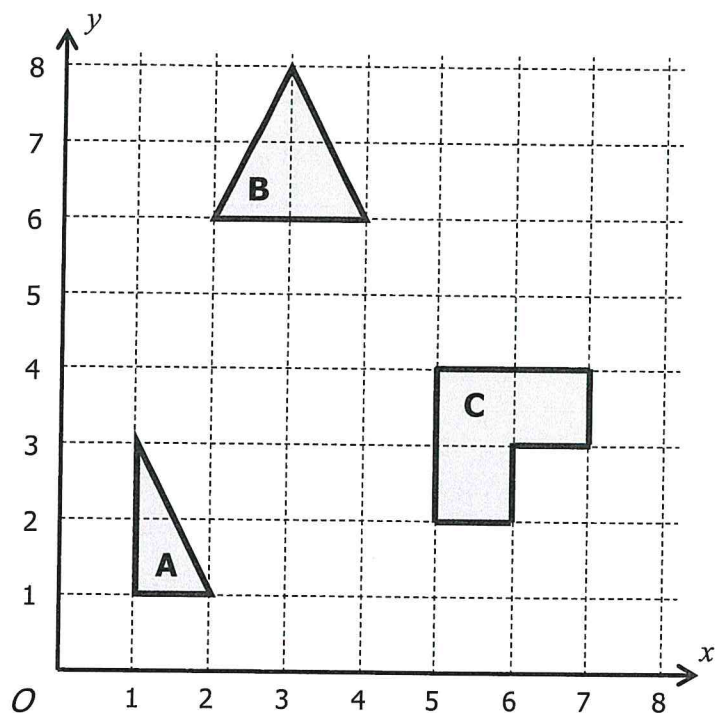


Diagram 2

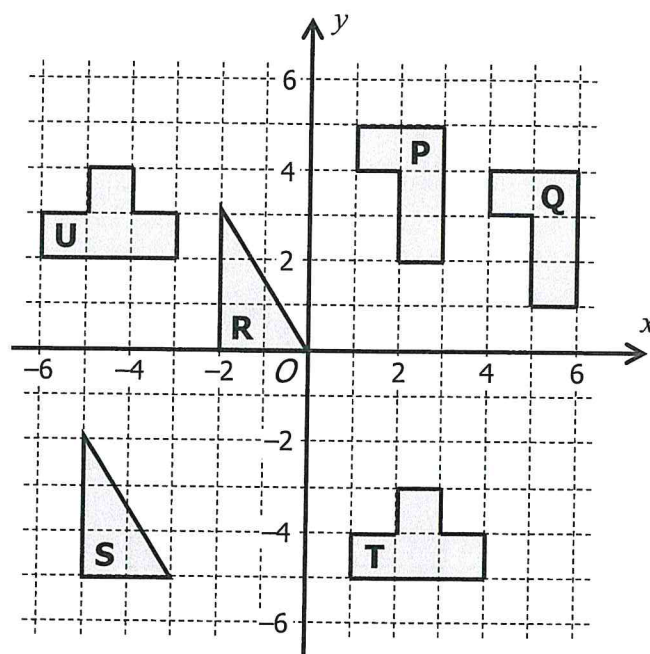


Diagram 3

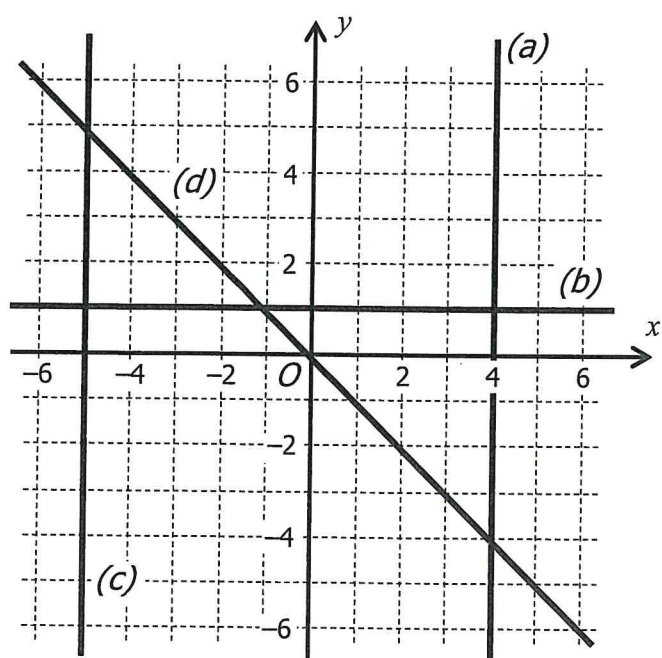


Diagram 4

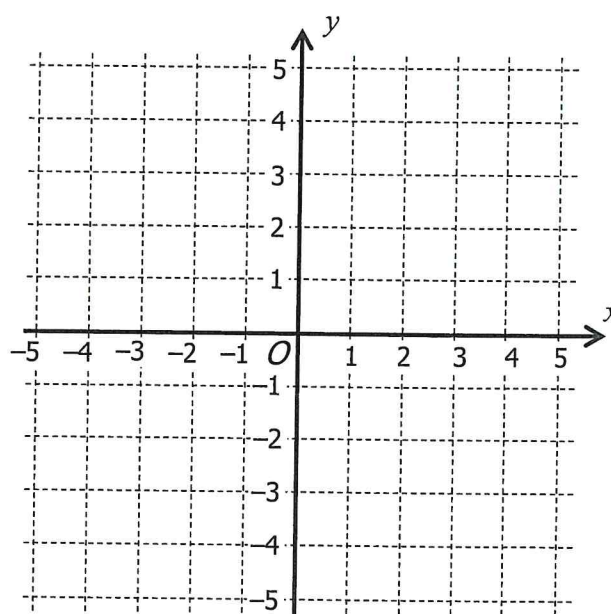


Diagram 5

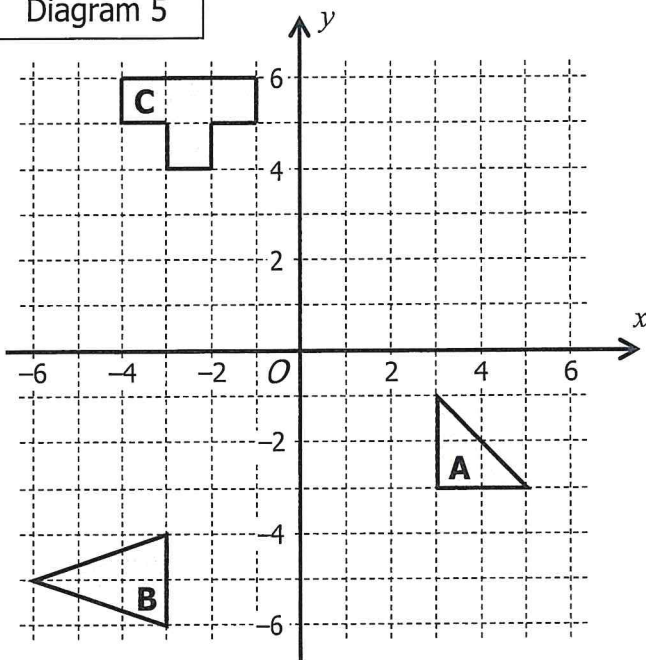


Diagram 6

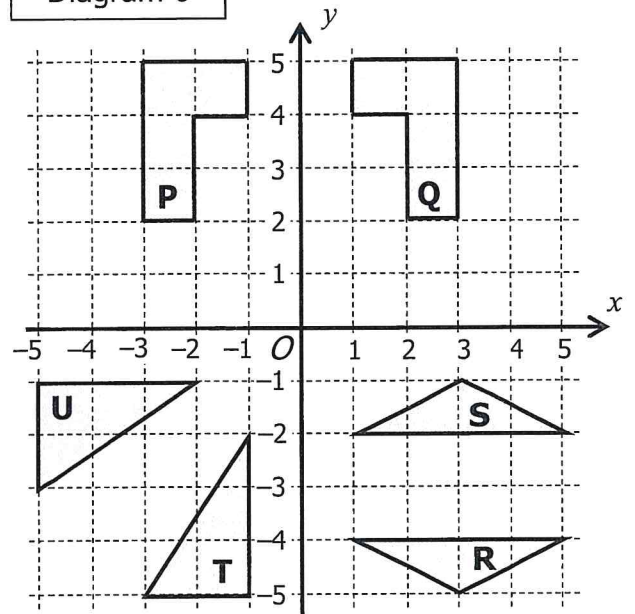


Diagram 7

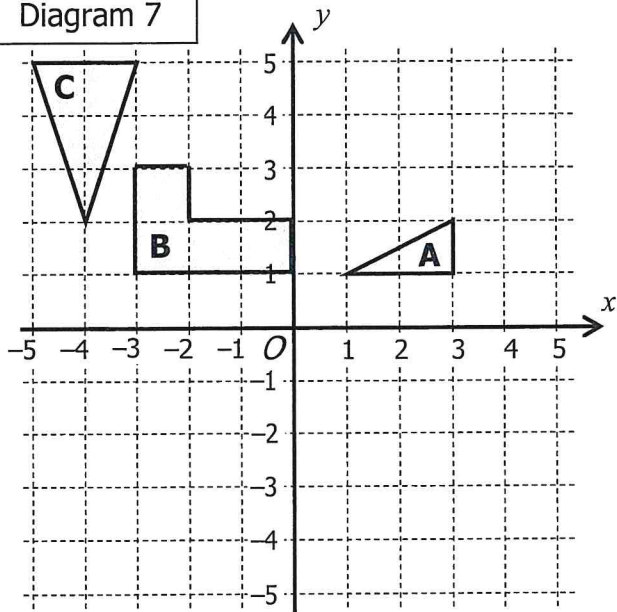


Diagram 8

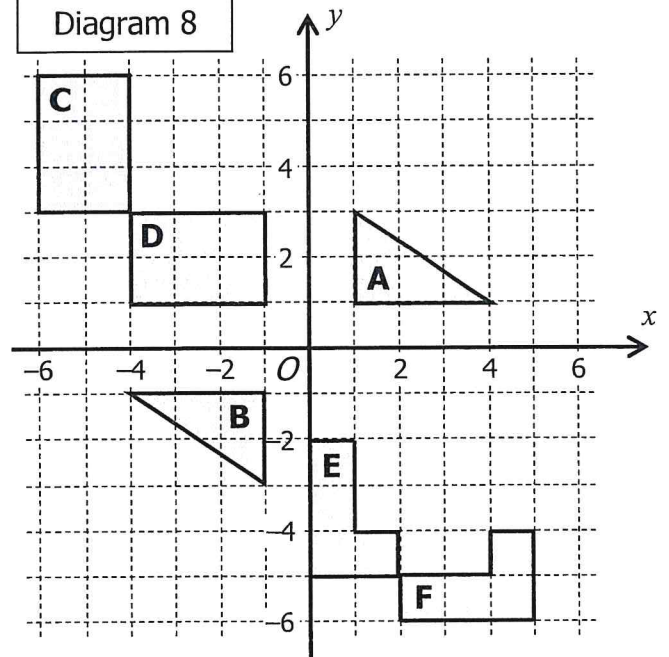


Diagram 9

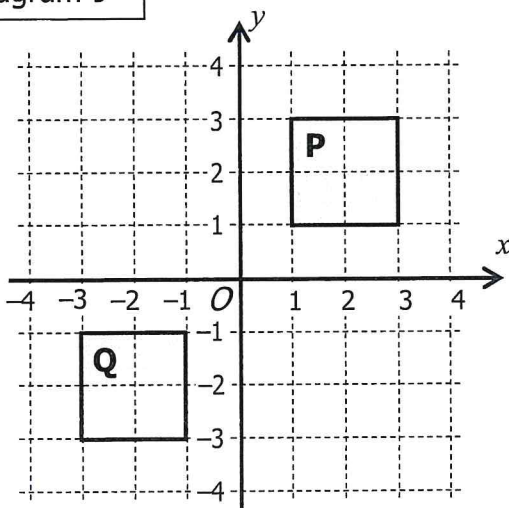
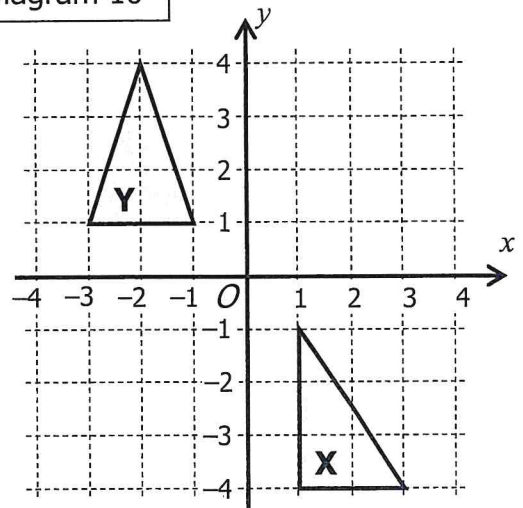



Diagram 10

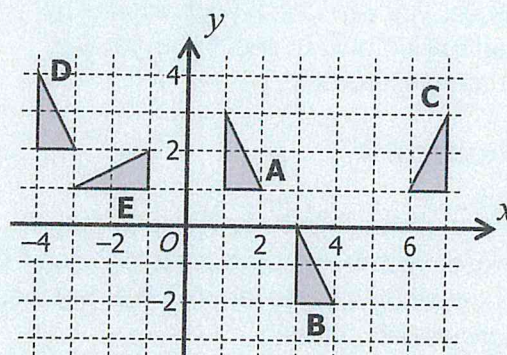


Quick Quiz

- Write these numbers in ascending order:
4.064 4.6 4.406 4.06
- Draw a quadrilateral containing a reflex angle.
- On this probability scale, mark the event 'rolling a 3 on a fair dice':

- How many centimetres are in a metre?
- What is the formula for the area of a parallelogram?

Review of Session 7

Describe each transformation fully.



- | | |
|---------------------|---------------------|
| (1) A onto B | (2) A onto C |
| (3) A onto D | (4) A onto E |

The focus for today's session is... Expressions in algebra

Reminder

Example

Multiply out the expression $3(a + 2b)$.

Solution

The '3' multiplies both terms inside the bracket:

$$\begin{array}{c}
 \times \\
 \curvearrowright \\
 3(a + 2b) \\
 \curvearrowleft \\
 \times
 \end{array}$$

Answer: $3a + 6b$

Example

Multiply out $2x(3x - 4y)$.

Solution

The '2x' multiplies both terms inside the bracket.

Answer: $6x^2 - 8xy$

Practice Questions

- Multiply out the following:
 - $2(p + 1)$
 - $4(t - 3)$
 - $5(y + 10)$
 - $3(x - 5y)$
 - $a(a - b)$
 - $w(2w + 5)$
- Multiply out the following:
 - $3k(k + 2m)$
 - $2x(x + 3y)$
 - $5p(2p - 5q)$
 - $6s(3u + 2v)$
 - $xy(3x - 5y)$
 - $4ab(3a + 10b)$

Reminder

Factorising is the opposite process to multiplying out.

Example 1

$$9s + 15t = 3(3s + 5t)$$

(Note: you can check your answer by multiplying out, to see if you get the original expression.)

Example 2

$$4x^2 - 6xy = 2x(2x - 3y)$$

(Note: you should always factorise as fully as possible. An answer of $x(4x - 6y)$ would not get full marks.)

Practice Questions

(3) Factorise these expressions:

(a) $2a + 6$

(b) $15p - 5q$

(c) $18 + 24x$

(d) $14u + 21v$

(4) Factorise these expressions:

(a) $x^2 - 2x$

(b) $2ab + 4b^2$

(c) $10m^2 + 15mn$

(d) $xy^2 + 2x^2y$

Reminder

When you multiply powers, you **add** the indices:

$$x^6 \times x^5 = x^{11}$$

When you divide powers, you **subtract** the indices:

$$\frac{t^{10}}{t^3} = t^7$$

When one power is raised to another power, you **multiply** the indices:

$$(a^3)^4 = a^{12}$$

Practice Questions

(5) Simplify these expressions:

(a) $a^4 \times a^2$ (b) $(x^2)^5$

(c) $\frac{y^9}{y^5}$ (d) $h \times h^8$

(e) $p^{12} \div p^4$ (f) $(s^5)^4$

(6) Simplify these expressions:

(a) $\frac{n^{12} \times n^6}{n^9}$

(b) $(4x^3)^2$

Reasoning and Problem-Solving

(7) Both of these contain a mistake. Explain what mistake has been made, and correct it.

(a) $2(x + 8) = 2x + 10$

(b) $\frac{m^6}{m^3} = m^2$

(8) (a) Multiply out and simplify the expression $2(m + n) + 3(m + 6n)$.

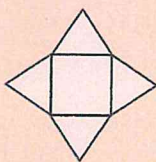
(b) Factorise your answer to (a).

(c) m and n are positive integers

Explain why $2(m + n) + 3(m + 6n)$ is always a multiple of 5.

Quick Quiz

- (1) Which of these numbers is not a cube number?
1 12 64 125
- (2) Work out $2,172 \div 4$ without a calculator.
- (3) If $p = 1$ and $q = 7$, work out the value of $p^2 + q^2$.
- (4) This net can be folded into a solid shape. What is the mathematical name for the shape?
- (5) Tamsin and Alison share some money in the ratio 3 : 7. What fraction of the money does Tamsin receive?



Review of Session 8

- (1) Multiply out:
 - (a) $3(m - 5)$
 - (b) $x(2x + 5y)$
 - (c) $5t(4s - 5t)$
- (2) Factorise as fully as possible:
 - (a) $2a + 8b$
 - (b) $u^2 - 3u$
 - (c) $6x^2y + 10xy^2$
- (3) Simplify:
 - (a) $x^8 \times x^5$
 - (b) $(n^3)^5$
 - (c) $\frac{p^{20}}{p^4 \times p^6}$

The focus for today's session is... Rounding and Estimation

Reminder

Example

Round 48.29 to the nearest whole number.

Solution

Answer = 48

The '2' means it is nearer to **48** than 49.

Example

Round 5.376 to 1 decimal place.

Solution

Answer = 5.4

The '7' means it is nearer to **5.4** than 5.3.

When the next number is 5 or above, round UP.

Practice Questions

- (1) Round to the nearest whole number:
 - (a) 67.82
 - (b) 401.52
- (2) Round to 1 decimal place:
 - (a) 1.528
 - (b) 17.89
- (3) Round to 2 decimal places:
 - (a) 0.7821
 - (b) 2.1067
- (4)
 - (a) Round 13,572 to the nearest hundred.
 - (b) Round 697 to the nearest ten.
 - (c) Round 1,783,478 to the nearest thousand.

Reminder

When rounding using significant figures:

- The first significant figure is the first non-zero number
- The answer will be close to the number you are rounding
- Use the usual rules about rounding up and down

So, for example:

0.00582 to 1 s.f. is 0.006 (not 0)
1,742 to 2 s.f. is 1,700 (not 17)
2.09571 to 3 s.f. is 2.10 (not 2.1 or 2.096)

Practice Questions

- (5) Round to 1 significant figure:
(a) 6.3 (b) 18.1
(c) 0.0457 (d) 13,457
- (6) Round to 2 significant figures:
(a) 0.007418 (b) 24.85
(c) 1752.1 (d) 5.0368
- (7) Round to 3 significant figures:
(a) 4,763,700 (b) 0.01085
(c) 2.47901 (d) 13,108

Reminder

When estimating the answer to a **multiplication**, round the numbers to 1 significant figure.

Example

$$578 \times 0.416 \approx 600 \times 0.4 = \underline{240}$$

When estimating the answer to a **division**, round to the nearest 'easy' numbers.

Examples

$$4631 \div 51 \approx 4500 \div 50 = \underline{90}$$

(rounded to 4,500 as 45 is a multiple of 5)

$$230.8 \div 0.68 \approx 210 \div 0.7 = \underline{300}$$

(rounded to 210 as it is a multiple of 7.)


Practice Questions

- (8) Estimate the answers to these calculations:
- (a) 52.8×3.871
- (b) 947×41.9
- (c) $3641 \div 67$
- (d) $\frac{4,178}{7.68}$
- (e) 0.237×84.01
- (f) $\frac{153}{0.041}$

Reasoning and Problem-Solving

- (9) A number is 17.5 when rounded to 1 decimal place, and 17 when rounded to the nearest whole number.
What could the number be?
- (10) Tickets for a concert cost £27.95.
The total income from ticket sales was £18,950.10.
Estimate the number of people who bought tickets.

Quick Quiz

- (1) The temperature in Glasgow at midday is 13 degrees higher than it was at midnight. At midnight it was -4°C ; what was the temperature at midday?
- (2) List all the factors of 24.
- (3) What is the order of rotation symmetry of a square? 
- (4) Measure the angle a at the top of the Answer Sheet.
- (5) The numbers 1 to 12 are put into a bag, and one is drawn at random. What is the probability it is prime?

Review of Session 9

- (1) Round:
 - (a) 5,483 to the nearest hundred
 - (b) 13,654.3 to the nearest ten
 - (c) 45.807 to the nearest whole number
 - (d) 823 to 1 significant figure
 - (e) 7.4601 to 1 decimal place
 - (f) 15.1642 to 2 decimal places
 - (g) 0.058429 to 3 significant figures
- (2) Estimate the answers to these:
 - (a) 562.7×17.8
 - (b) $\frac{621}{8.2}$
 - (c) $\frac{1,134}{0.573}$

The focus for today's session is...

Pie Charts

Reminder

To draw a pie chart, work out how many degrees represent each person.

In Questions (1) and (2), this has already been done for you on the Answer Sheet.

In Diagram 1, the circle has been marked out into 9 equal parts. Each part of the circle represents one person in the survey.

Diagram 2 has been marked out into 20 parts, one for each person in the survey.

Practice Questions

- (1) Use the pie chart in **Diagram 1** on the Answer Sheet to represent the following data.

Favourite sport	People
Football	6
Tennis	2
Rugby	1

- (2) Use the pie chart in **Diagram 2** to represent the following data.

Transport to school	People
Walk	5
Cycle	7
Car	1
Bus	4
Train	3

Reminder

To work out the angles in a pie chart:

- Step 1: Add up the numbers in each category to get a total;
- Step 2: Divide 360° by this total – this will tell you how many degrees represent each person;
- Step 3: Multiply this by the number for each category – this will give the angles you need to draw;
- Step 4: Add up the angles, to check the total is 360° .

Practice Questions

- (3) (a) Work out the pie chart angles for these data:

Grades in mock	Students
Grade 5	3
Grade 4	8
Grade 3	7

- (b) Draw the pie chart on **Diagram 3**.

- (4) Draw a pie chart for these data on **Diagram 4**.

Football team supported	People
Liverpool	11
Chelsea	6
Arsenal	8
Man Utd	15

Reminder

For the questions opposite, you will need a calculator.

Use the same method as before. Here is some advice to help with accuracy.

- Step 2: Write down the answer to at least 4 significant figures
- Step 3: Use the accurate value from Step 2 each time, but round the answer to the nearest degree
- Step 4: Don't worry if they add up to 359° or 361° . (It'll be close enough!)

Practice Questions

- (5) Draw a pie chart for these data on **Diagram 5**.

Dietary requirements	People
Vegetarian	3
Vegan	5
Pescatarian	8
None of the above	26

- (6) Draw a pie chart for these data on **Diagram 6**.

Options chosen	Students
French	45
Geography	70
History	58
Media	19
PE	25

Reasoning and Problem-Solving

- (7) **Diagram 7** shows an accurate pie chart concerning marital status. 60 people were questioned in the survey.

Work out the number of people in each category, and write them in the table.

- (8) **Diagram 8** shows two pie charts. They show the types of car in two groups.

There were more electric cars in Group A than in Group B.

Is this statement true? Explain your answer.

ANSWER SHEET FOR SESSION 10

Diagram 1

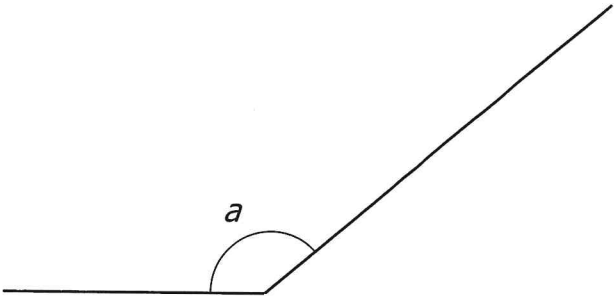
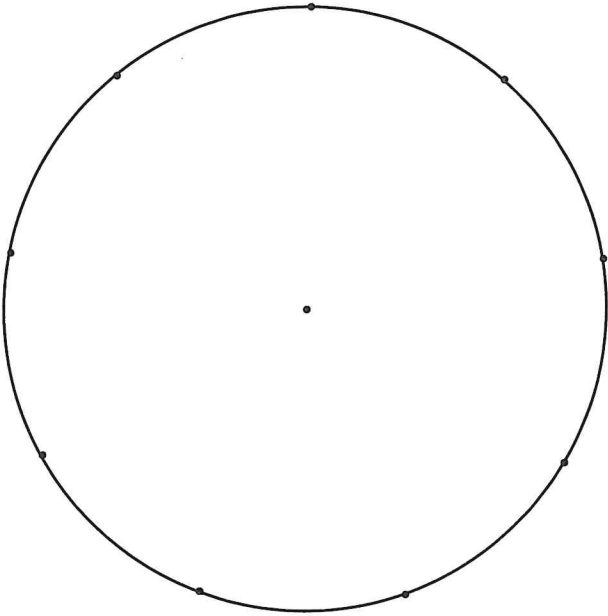


Diagram 2

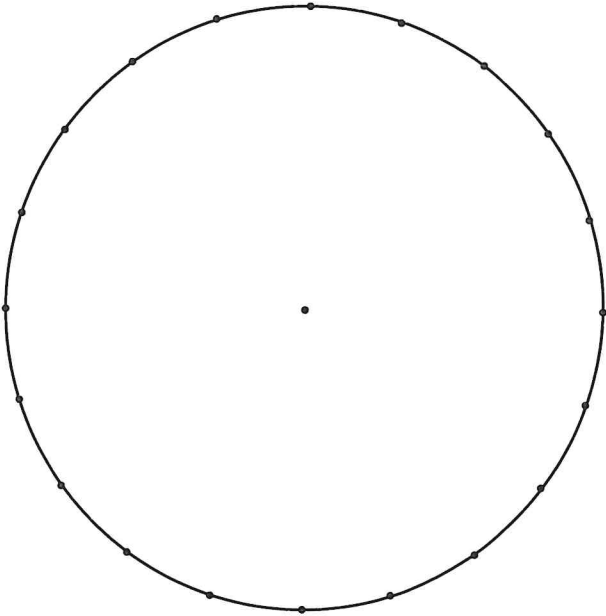


Diagram 3

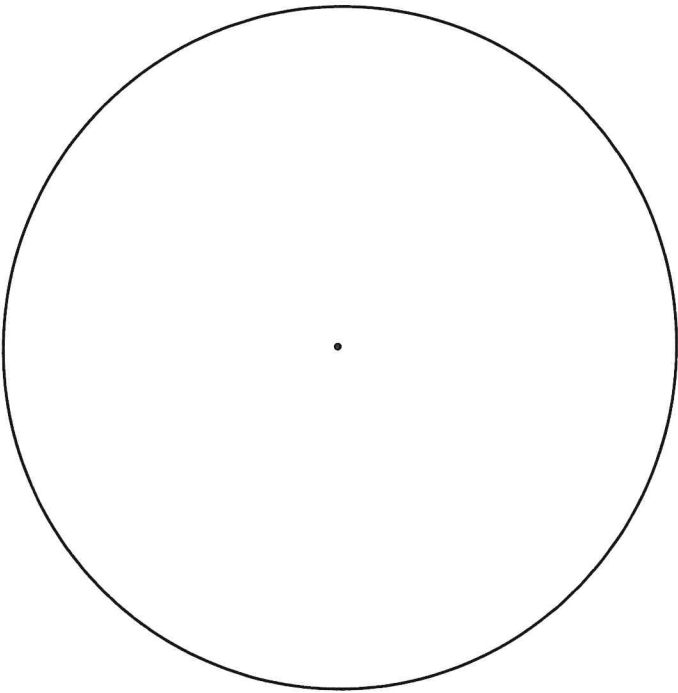


Diagram 4

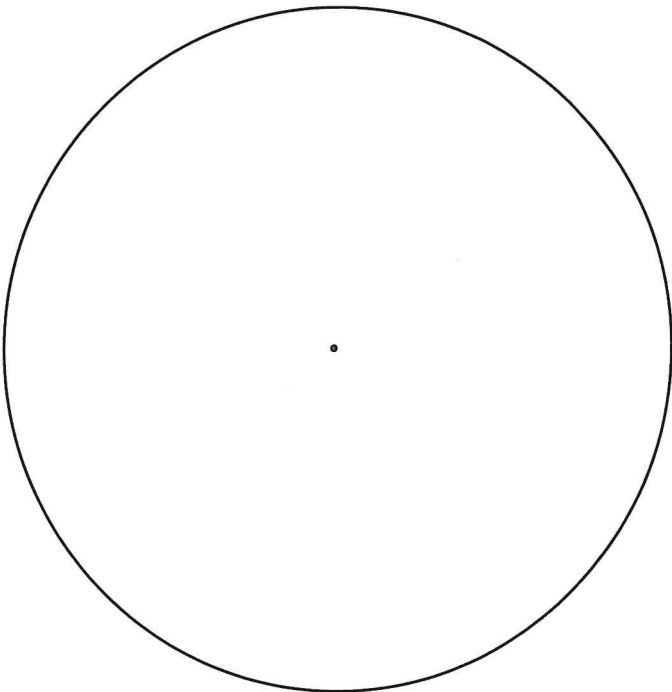


Diagram 5

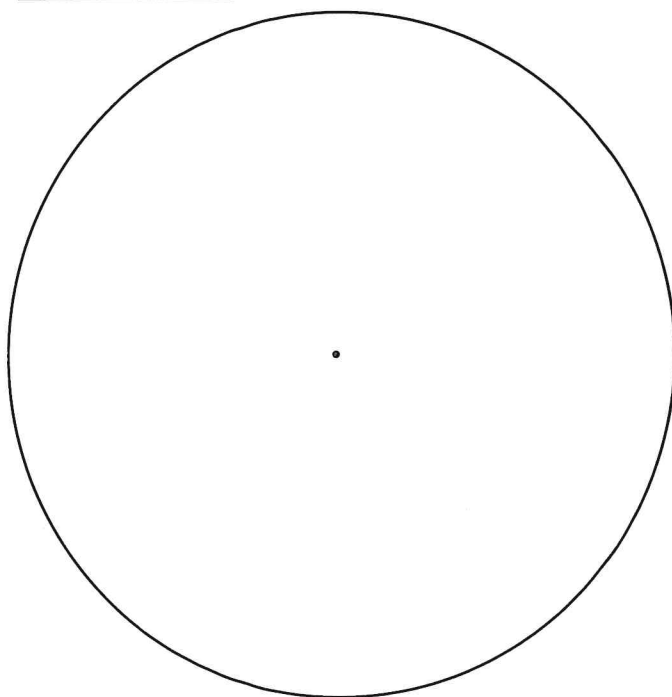


Diagram 6

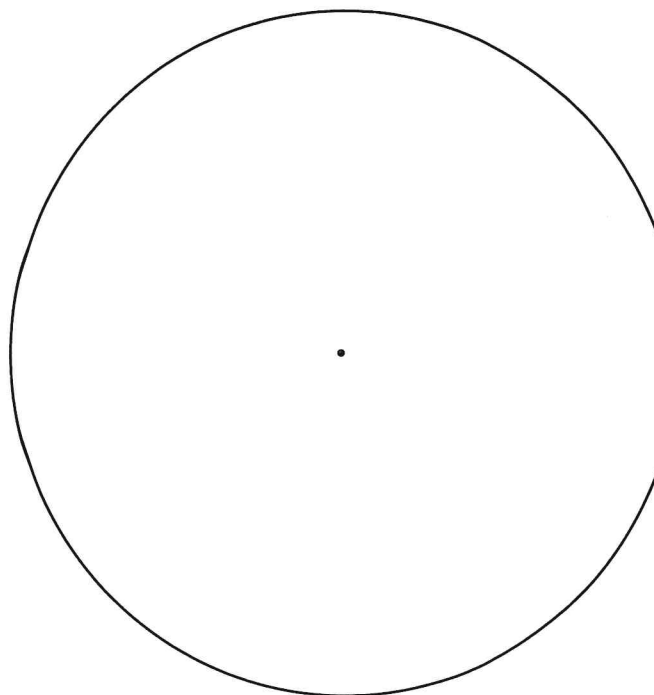


Diagram 7

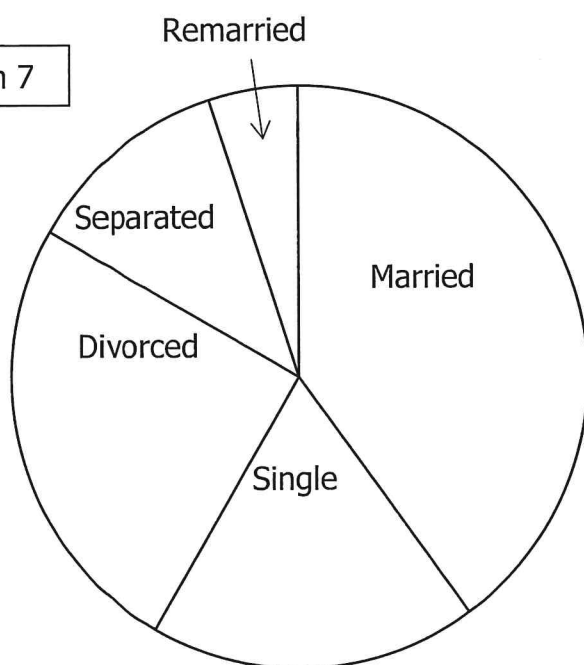
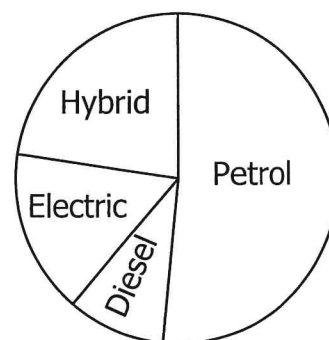
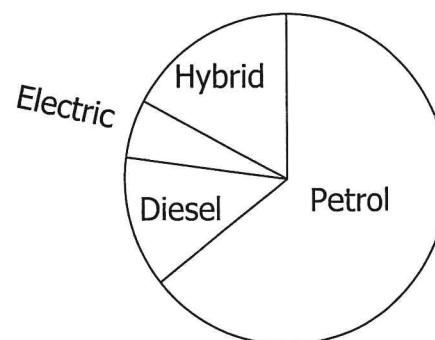


Diagram 8

Group A



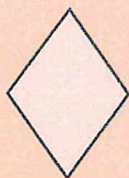
Group B



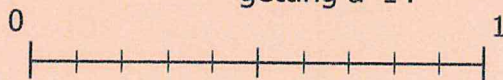
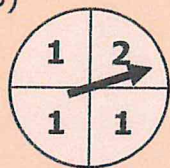
Marital status	People
Married	
Single	
Divorced	
Separated	
Remarried	

Quick Quiz

- (1) Work out 9×329 without a calculator.
- (2) What is the value of $\sqrt[3]{8}$?
- (3) Draw all the lines of symmetry on this rhombus.
- (4) Work out $(-5) + (-3)$ without a calculator.



- (5) This spinner is spun. On the probability scale, mark the probability of getting a '1'.



Review of Session 10

- (1) Suki plays four instruments. The table shows how long she spent playing each one last week.

	Time (hours)
Piano	8
Guitar	4
Violin	1
Clarinet	5

She puts this information in a pie chart. Work out the angle for each instrument.

- (2) Rachel is drawing a pie chart to show the favourite animals of the children in her class.

6 of the class said their favourite animal was a chicken. The angle in the pie chart for 'chicken' was 72° .

How many children are in the class?

The focus for today's session is... Equations (Part 1)

Reminder

For a one-step equation, remember your 'inverse operations':

- Addition is the opposite of subtraction (and *vice versa*)
- Multiplication is the opposite of division (and *vice versa*)

Example

$$t - 18 = 53$$

Add 18: $t = 53 + 18 = \underline{71}$

Example

$$5x = 28$$

Divide by 5: $x = \frac{28}{5} = \underline{5.6}$

Practice Questions

- (1) Solve these equations.

(a) $p + 15 = 27$

(b) $4n = 36$

(c) $\frac{w}{6} = 12$

(d) $x - 9 = 32$

(e) $18y = 81$

(f) $\frac{u}{24} = 7$

(g) $a + 68 = 250$

(h) $t - 0.52 = 1.7$

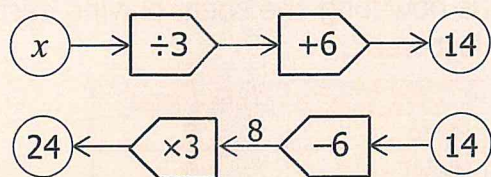
(i) $6h = 3$

Reminder

Example

$$\frac{x}{3} + 6 = 14$$

One way is to use a flow diagram:



so $x = 24$.

Another way is to 'balance' the equation. You must do the same thing to both sides of the equation:

$$\begin{array}{ccc} \frac{x}{3} + 6 = 14 & & \\ \textcircled{-6} & & \textcircled{-6} \\ \frac{x}{3} = 8 & & \\ \textcircled{\times 3} & & \textcircled{\times 3} \\ x = 24 & & \end{array}$$

Practice Questions

(2) Solve these equations.

(a) $4n + 2 = 46$

(b) $\frac{x}{2} - 3 = 8$

(c) $5(y - 2) = 35$

(d) $12a - 3 = 75$

(e) $\frac{m + 5}{4} = 9$

(f) $7p + 9 = 100$

(g) $\frac{w}{4} - 31 = 17$

(h) $15(k + 2) = 105$

(3) Solve these equations.
(They are slightly harder.)

(a) $\frac{2x - 9}{5} = 3$

(b) $\frac{5(y + 4)}{6} = 15$

Using an equation to solve a problem

Kelly is k years old.

Lucy is 3 years younger than Kelly.

Maisie is twice as old as Kelly.

Their total age is 73.

How old is Kelly?

Solution

Lucy is $(k - 3)$ years old. Maisie is $2k$.

So: $k + (k - 3) + 2k = 73$

Simplify: $4k - 3 = 73$

Solve: $k = 19$

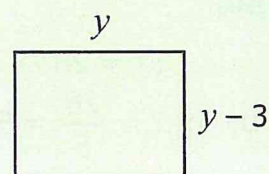
Reasoning and Problem-Solving

- (4) In a bag there are green, red and white counters.
There are x green counters.
There are 3 times as many red counters as green counters.
There are 10 more white counters than green counters.

There are 45 counters in total.

How many green counters are there?

- (5) The perimeter of this rectangle is 28 cm.
Work out the value of y .



Quick Quiz

- (1) Write these in ascending order:
1.055 1.55 1.505 1.05
- (2) What is the missing number in this sequence?
1 4 16 25 ...
- (3) Work out $42 \div (-6)$ without a calculator.
- (4) Draw a quadrilateral containing exactly two obtuse angles.
- (5) 6 one-litre cartons of milk cost £5.10. What is the cost of 8 one-litre cartons?

Review of Session 11

- (1) Solve these equations.

(a) $x + 16 = 50$

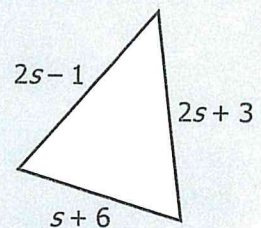
(b) $\frac{t}{4} = 8$

(c) $3(k - 7) = 24$

(d) $\frac{n}{6} + 11 = 36$

- (2) The perimeter of this triangle is 30 cm.

Work out the value of s .



The focus for today's session is...

Fractions, decimals and percentages

Reminder

Percentages are fractions of 100.

To convert decimals to percentages:

$$0.56 = \frac{56}{100} = \underline{56\%} \quad 0.3 = \frac{30}{100} = \underline{30\%}$$

Fractions can be converted to percentages when the denominator is a factor of 100:

$$\frac{7}{25} \xrightarrow{\times 4} = \frac{28}{100} = \underline{28\%}$$

Sometimes you need to simplify the fraction first:

$$\frac{38}{40} \xrightarrow{\div 2} = \frac{19}{20} \xrightarrow{\times 5} = \frac{95}{100} = \underline{95\%}$$

Practice Questions



- (1) Convert these decimals to percentages:

(a) 0.45 (b) 0.8

(c) 0.02 (d) 0.99

(e) 0.125 (f) 0.647

- (2) Convert these fractions to percentages

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$

(c) $\frac{9}{10}$ (d) $\frac{9}{20}$

(e) $\frac{31}{50}$ (f) $\frac{42}{60}$

(g) $\frac{28}{80}$ (h) $\frac{84}{200}$

Reminder

Percentages are useful in describing situations.

Example

50 people go on an outing.
37 of them paid in advance.

What percentage paid in advance?

Solution

$$37 \text{ out of } 50 = \frac{37}{50} \xrightarrow{\times 2} = \frac{74}{100} \xrightarrow{\times 2} = \underline{74\%}$$

Practice Questions



- (3) In a survey of 25 people, 17 said they liked ice cream.

What percentage is this?

- (4) Jake went to a football match with £50 spending money. He spent £38 on food.

What percentage of his spending money did Jake spend on food?

- (5) A class contains 7 boys and 13 girls.

What percentage are boys?

Reminder

With more difficult numbers, use a calculator to work out the percentage.

Use the fact that a fraction is the same as division:

$\frac{36}{79}$ is the same as $36 \div 79$

Example

Convert $\frac{11}{46}$ to a percentage, to the nearest whole number.

Solution

$$\frac{11}{46} = 11 \div 46 = 0.239 \dots$$

This is 23.9%, so answer = 24%

Practice Questions

- (6) Convert these to percentages, to the nearest whole number

$$(a) \frac{1}{7} \qquad (b) \frac{15}{19}$$

$$(c) \frac{23}{85} \qquad (d) \frac{137}{160}$$

- (7) It cost Hamza £7.70 to make a cake. When he sold it, he made £2.29 profit.

Work out the profit as a percentage of the cost, correct to 1 decimal place.

- (8) In the Autumn Term, there were 73 days of school. Anna was absent for 16 of these.

What was Anna's attendance, as a percentage, correct to 1 d.p.?

Reasoning and Problem-Solving

- (9) At Appledown School, 45 students out of 57 passed a computing test. At Beckwith School, the same test was passed by 95 students out of 125.

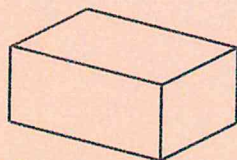
Which school did better in the test? Show your working clearly.

- (10) Jeremy uses $\frac{1}{2}$ of his allotment to grow potatoes, $\frac{3}{20}$ to grow beans and $\frac{8}{25}$ for carrots.

What percentage of Jeremy's allotment is left?

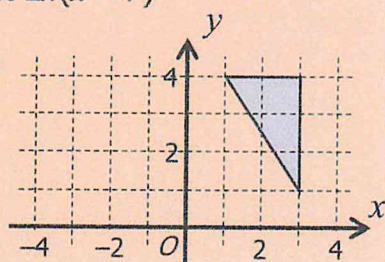
Quick Quiz

- (1) Work out $4^3 + 5^3$ without a calculator.
- (2) Convert 3.5 km to metres.
- (3) How many edges are there on a cuboid?



- (4) Multiply out $2x(x - 7)$

- (5) Reflect the triangle in the y -axis.



Review of Session 12

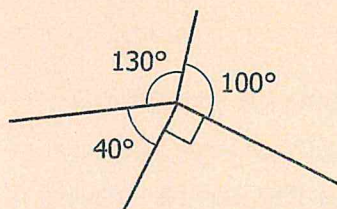
- (1) Convert 0.23 to a percentage.
- (2) Convert 58% to a decimal.
- (3) What is 9% as a decimal?
- (4) Without a calculator, convert the fraction $\frac{21}{25}$ to a percentage.
- (5) Use a calculator to convert the fraction $\frac{5}{78}$ to a percentage.
- (6) In a class election, 19 people voted for Danny and 8 people voted for Sara. What percentage voted for Sara?

The focus for today's session is...

Angles

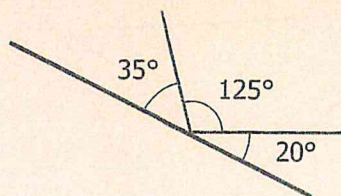
Reminder

The sum of angles **round a point** is **360°** .



$$40^\circ + 130^\circ + 100^\circ + 90^\circ = 360^\circ$$

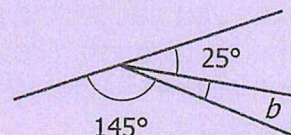
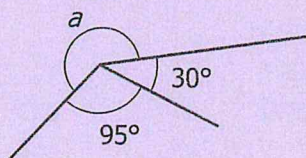
The sum of angles **at a point on a straight line** is **180°** .



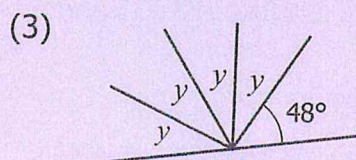
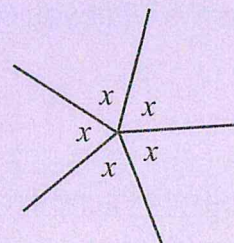
$$35^\circ + 125^\circ + 20^\circ = 180^\circ$$

Practice Questions

- (1) Work out angles a and b in these diagrams.



- (2) The angles marked x are all equal.
Work out the value of x .

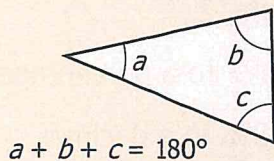


The angles marked y are all equal.

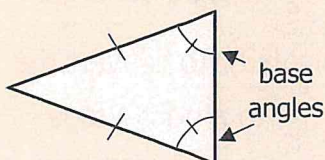
Work out the value of y .

Reminder

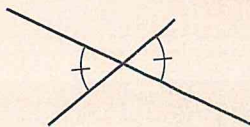
The sum of the interior angles of a triangle is 180° .



In an **isosceles triangle**, two sides are the same length. The two base angles are equal.



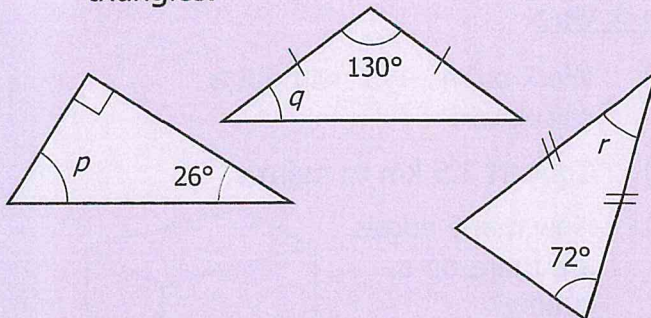
Where straight lines cross, opposite angles are equal.



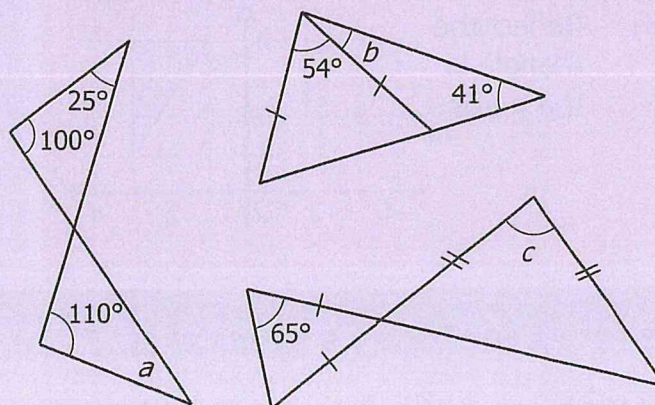
(These angles are often called '**vertically opposite**'.)

Practice Questions

- (4) Work out the angles p , q and r in these triangles.



- (5) Work out the angles a , b and c in these diagrams.

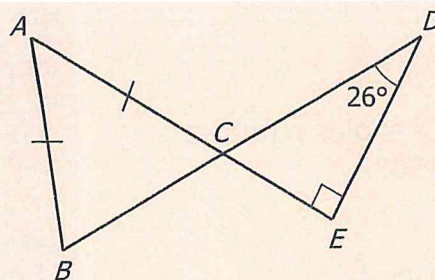


Reminder

In the exam, you may be asked to give reasons.

Suppose you had to find angle BAC in this diagram. (You are given that the lengths AB and AC are equal.)

Here is what you could write:



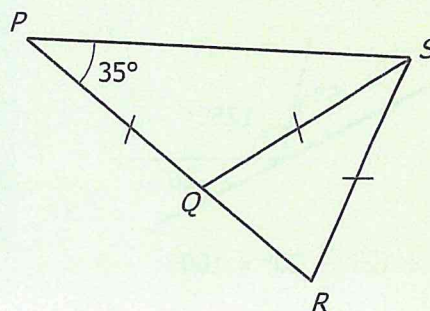
$DCE = 64^\circ$	Angles in a triangle add up to 180° .
$ACB = 64^\circ$	Vertically opposite angles are equal.
$ABC = 64^\circ$	Base angles of an isosceles triangle are equal.
$BAC = 52^\circ$	Angles in a triangle add up to 180° .

Reasoning and Problem-Solving

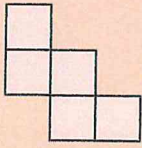
- (6) In this diagram, the lengths PQ , QS and RS are all equal.

Work out angle QSR .

Show how you got your answer, giving reasons for each stage of your working.

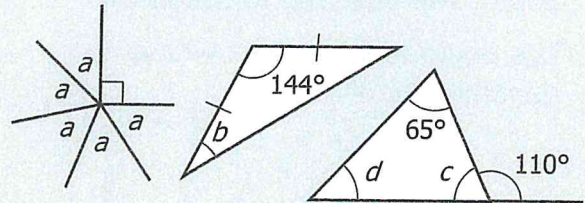


Quick Quiz

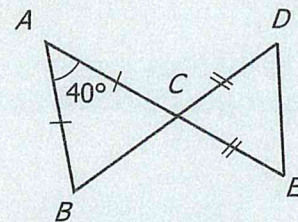
- Work out the value of $10 - 3 \times 2$
- Round the number 25,709 to 2 significant figures.
- Work out $3,402 \div 9$ without using a calculator.
- Here is the net of a cube with one square missing.  Where could the missing square go?
- $\frac{4}{5} = \frac{\square}{30}$ What number is missing here?

Review of Session 13

- Work out the angles marked with letters.



- Work out the size of angle CDE . Give reasons for each stage of your working




The focus for today's session is...

Adding and Subtracting Fractions

Reminder

When the denominators are the same, adding and subtracting are easy.

For example:

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$


Subtraction works the same way:

$$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

Practice Questions



- Work out these:

$$(a) \frac{3}{7} + \frac{2}{7} \quad (b) \frac{8}{9} - \frac{7}{9}$$

$$(c) \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$(d) \frac{10}{11} - \frac{6}{11} \quad (e) \frac{13}{15} - \frac{8}{15}$$

$$(f) \frac{63}{101} + \frac{5}{101} \quad (g) \frac{7}{10} - \frac{3}{10}$$

$$(h) \frac{6}{43} + \frac{11}{43} + \frac{23}{43}$$

Reminder

When the fractions have different denominators make the denominators the same.

Example: $\frac{3}{4} - \frac{1}{6}$

Find a number which is a multiple of 4 and 6. The best one to use is **12**.

Use equivalent fractions with a denominator of 12.

$$\frac{3}{4} = \frac{9}{12}$$

$$\frac{1}{6} = \frac{2}{12}$$

So: $\frac{9}{12} - \frac{2}{12} = \underline{\underline{\frac{7}{12}}}$

Practice Questions



(2) Work out these:

(a) $\frac{1}{2} + \frac{1}{3}$

(b) $\frac{2}{3} - \frac{1}{4}$

(c) $\frac{5}{6} - \frac{1}{9}$

(d) $\frac{1}{2} - \frac{3}{10}$

(e) $\frac{7}{12} + \frac{1}{3}$

(f) $\frac{14}{15} - \frac{5}{6}$

(g) $\frac{4}{7} + \frac{3}{8}$

(h) $\frac{7}{12} + \frac{3}{16}$

(i) $\frac{1}{2} + \frac{1}{3} + \frac{1}{8}$

(j) $\frac{3}{4} + \frac{1}{10} - \frac{2}{5}$

Reminder

When you have mixed numbers, convert them to **improper fractions** first.

Example

$$1\frac{1}{4} + 2\frac{2}{5}$$

Convert to improper fractions:

$$1\frac{1}{4} = \frac{5}{4}$$

$$2\frac{2}{5} = \frac{12}{5}$$

Then proceed as before:

$$\frac{5}{4} + \frac{12}{5} = \frac{25}{20} + \frac{48}{20} = \underline{\underline{\frac{73}{20}}}$$

(No need to change the answer to a mixed number unless you are asked to.)

Practice Questions



(3) Work out these:

(a) $1\frac{1}{3} - \frac{3}{5}$

(b) $\frac{1}{9} + 1\frac{1}{2}$

(c) $2\frac{3}{4} + \frac{7}{10}$

(d) $3\frac{2}{5} - 2\frac{1}{2}$

(e) $1\frac{5}{6} + 2\frac{1}{4}$

(f) $2\frac{1}{5} - 1\frac{3}{4}$

(g) $1\frac{3}{8} + \frac{5}{12}$

(h) $3\frac{1}{2} - 1\frac{2}{7}$

(i) $6\frac{2}{3} - 2\frac{8}{9}$

(j) $1\frac{5}{8} + 2\frac{3}{10}$

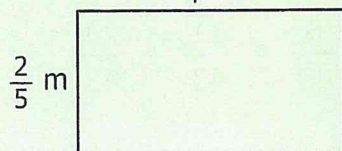
Reasoning and Problem-Solving

(4) Paul is working out $\frac{5}{7} + \frac{2}{3}$.

He makes a mistake in his method.
Explain the mistake he has made.

$$\frac{5}{7} + \frac{2}{3} = \frac{15}{21} + \frac{14}{21} = \underline{\underline{\frac{29}{42}}}$$

(5) $\frac{3}{4}$ m



Work out the perimeter of this rectangle.

Give your answer as a mixed number in its simplest form.

Quick Quiz

- (1) If $t = 5$, work out the value of $4t^2$.
- (2) Solve the equation
 $4(x - 7) = 18$
- (3) Without a calculator, convert $\frac{13}{20}$ to a percentage.
- (4) What is the value of $\sqrt{64}$?
- (5) How many faces are there on a square-based pyramid?

Review of Session 14

- (1) Work out these.
Give each answer in its simplest form.

(a) $\frac{1}{7} + \frac{3}{7}$	(b) $\frac{2}{3} - \frac{1}{6}$
(c) $\frac{7}{8} - \frac{5}{6}$	(d) $\frac{3}{4} + \frac{7}{15}$
(e) $1\frac{2}{5} + \frac{3}{8}$	(f) $4\frac{1}{6} - 1\frac{7}{10}$
- (2) Solve the equation
$$x + \frac{4}{7} = \frac{11}{12}$$

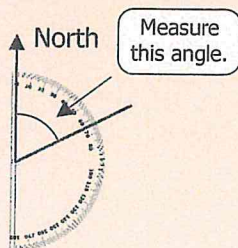
The focus for today's session is... Bearings and Loci

Reminder

A bearing is a **direction**.

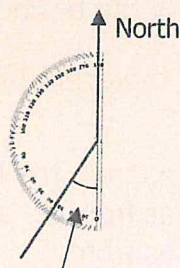
It is the angle measured **clockwise** from **North**.

Bearings always have **three figures**: so if the angle was 55° it would be written as 055° .



To measure a bearing with a protractor, make sure that zero points North.

If the angle is more than 180° , you will need to turn your protractor round, with zero pointing South.



Measure this angle and add 180° .

Practice Questions

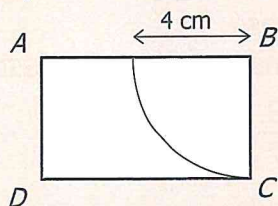
- (1) On **Diagram 1** on the Answer Sheet, measure the bearing from O of each of the six towns A to F .
- (2) What is the bearing of these directions?

(a) East	(b) South-West
(c) North-West	
- (3) (a) Draw the following journey on **Diagram 2**. Use a scale of 1 cm to 1 km.
 Start at P . Travel on a bearing of 070° for 5 km, to point Q .
 Then travel 6 km on a bearing of 215° to point R .
 (b) Measure the distance and bearing from P to R .

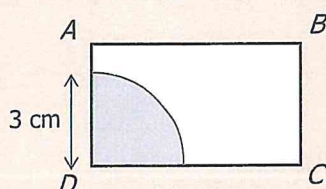
Reminder

Look carefully at these examples before trying Question (4).

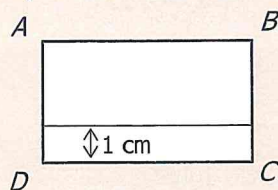
Exactly 4 cm from B .



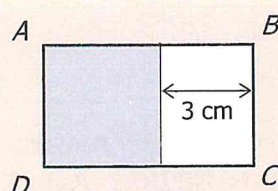
Less than 3 cm from D .



Exactly 1 cm from CD .



More than 3 cm from BC .



Practice Questions

You will need pencil, ruler and compasses.

- (4) On **Diagram 3**, there are four rectangles.

Show the points inside the rectangle which are:

- exactly 3 cm from A
- less than 4 cm from B
- exactly 2 cm from the side BC
- more than 3 cm from the side AD

- (5) On **Diagram 4**, there are four more rectangles.

Show the points inside the rectangle which are:

- less than 4 cm from P **and** less than 4 cm from Q
- less than 3 cm from PQ **and** less than 3 cm from QR
- more than 3 cm from S **and** less than 2 cm from RS
- more than 5 cm from P **and** more than 1 cm from RS .

Reasoning and Problem-Solving

- (6) **Diagram 5** is drawn to a scale of 1 cm to 1 km.

It shows two ships, S and T , at sea.

Another ship, U , is

- on a bearing of 015° from S ; and
- on a bearing of 335° from T .

By measuring these bearings, show accurately the position of U .

- (7) **Diagram 6** is the plan of a garden. S and T represent trees.

It is drawn to a scale of 1 cm to 2 m.

Grass grows:

- more than 2 metres from the boundary $ABCD$;
- more than 4 metres from S ; and
- more than 4 metres from T .

Shade the region where the grass grows.

ANSWER SHEET FOR SESSION 15

Diagram 1

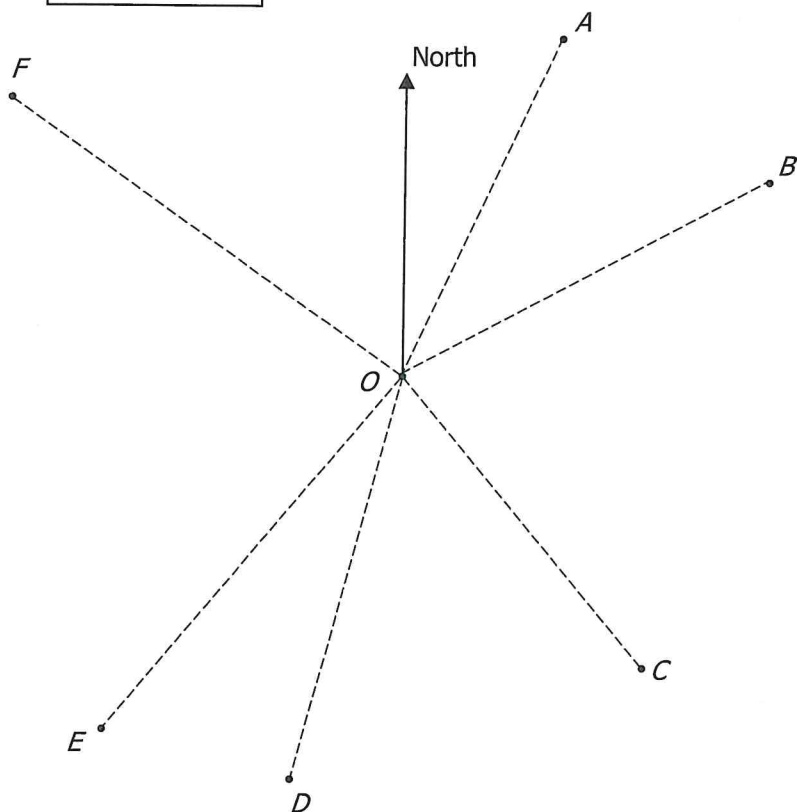


Diagram 2

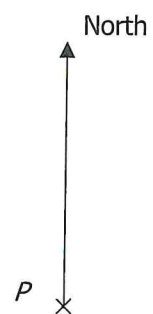


Diagram 3

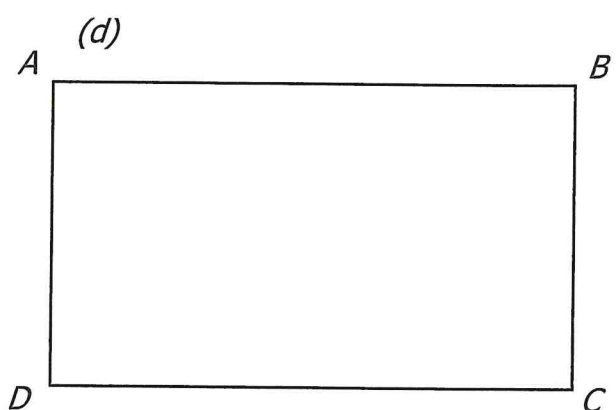
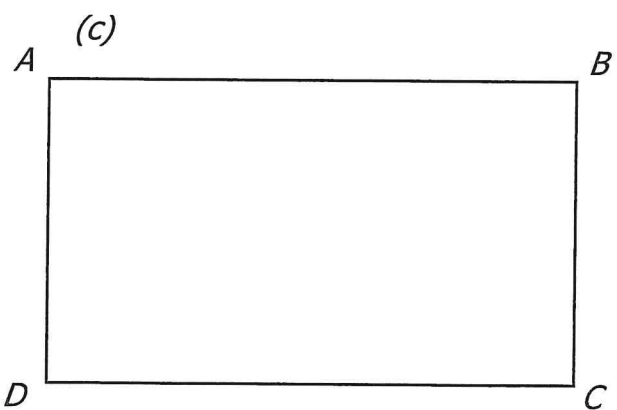
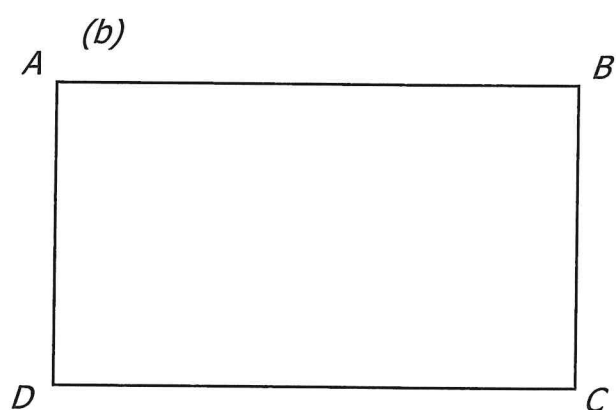
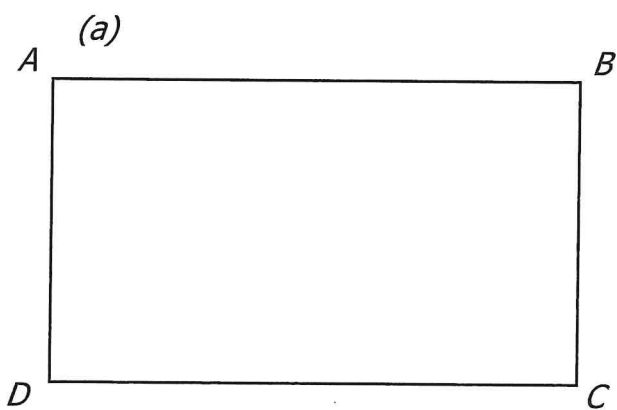


Diagram 4

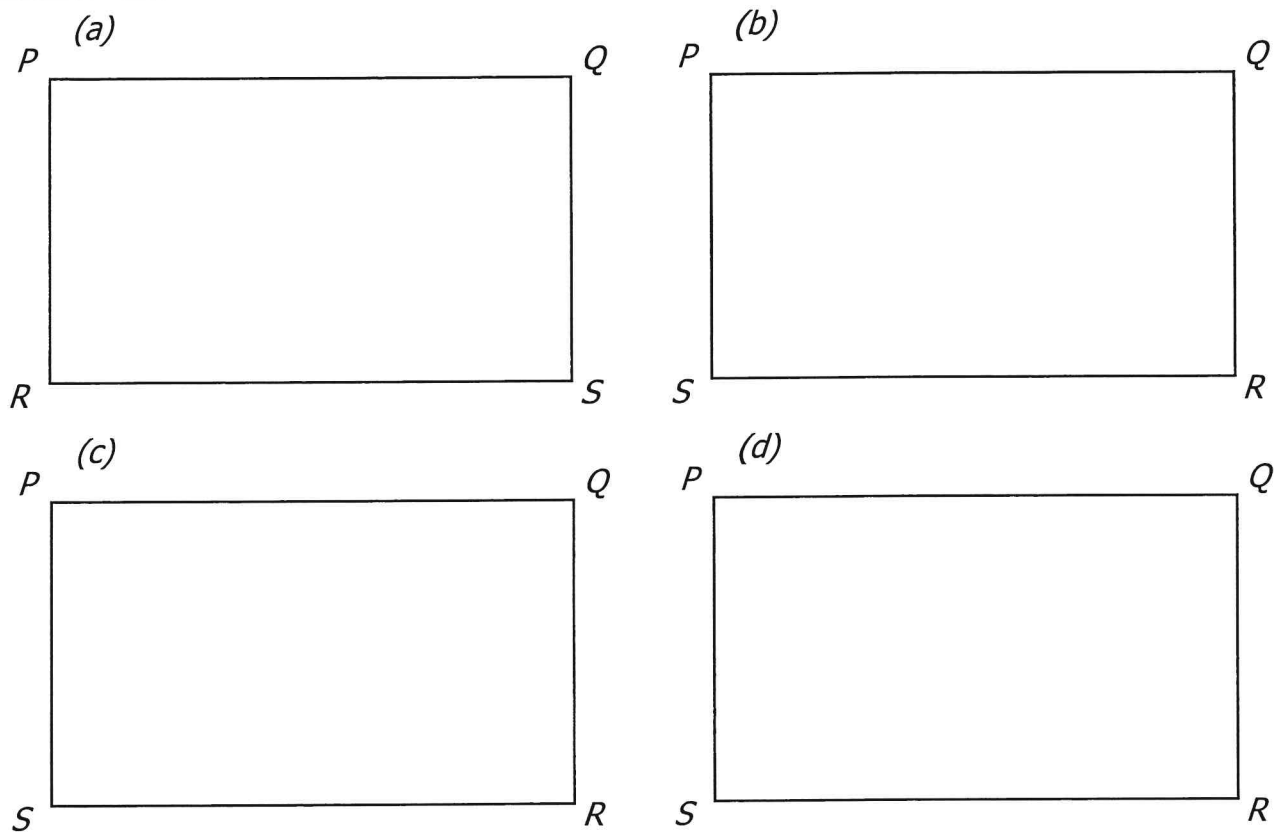
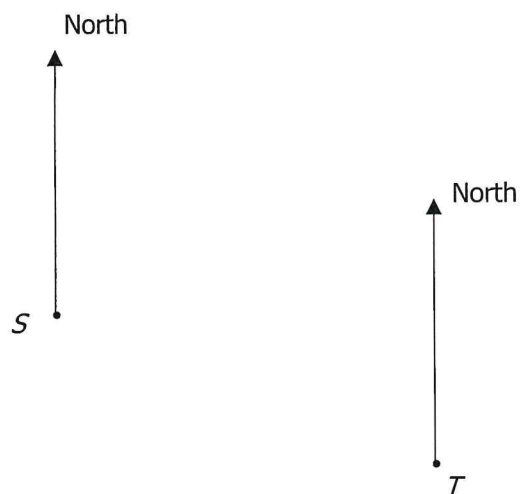
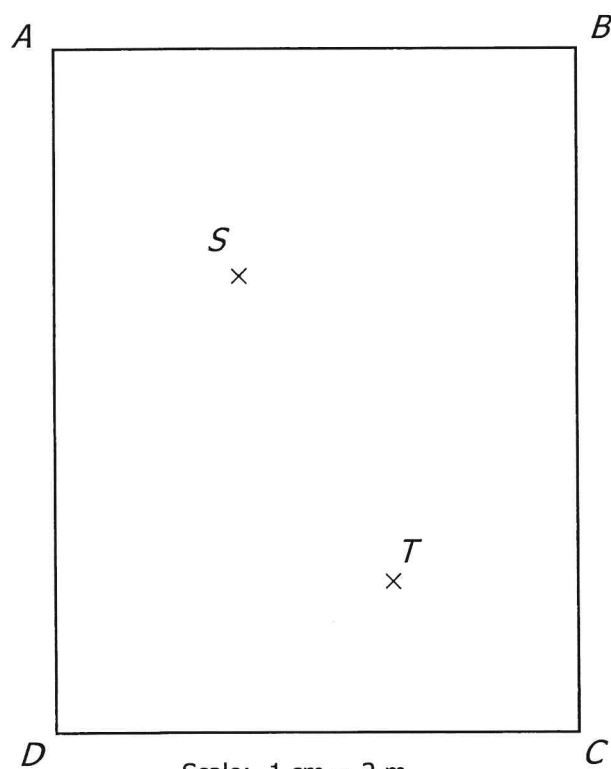


Diagram 5



Scale: 1 cm = 1 km

Diagram 6



Scale: 1 cm = 2 m

Quick Quiz

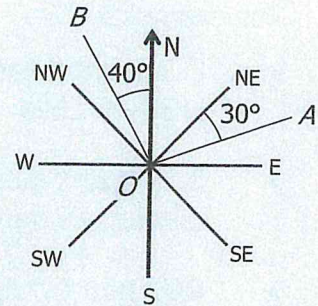
- Work out 772×8 , without a calculator.
- What is the formula for the area of a triangle?
- Work out $5 + 6 \times 1$.
- Here are the results of a survey on favourite sports.

Sport	No. of people
Football	13
Tennis	5
Cricket	2

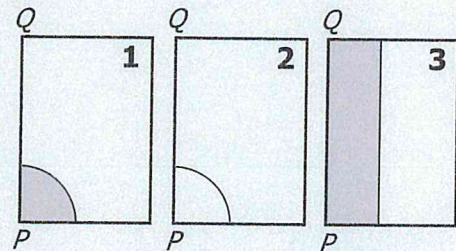
In a pie chart, what would be the angle for football?

Review of Session 15

- Work out the bearings of OA and OB in this diagram.
- Match the descriptions to the diagrams.



- A** exactly 2 cm from P
- B** less than 2 cm from PQ
- C** less than 2 cm from P



The focus for today's session is... Distance, speed and time

Reminder

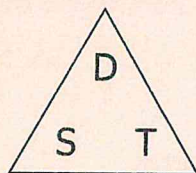
For simple problems, use these three formulas:

$$\text{distance} = \text{speed} \times \text{time}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

People often use this formula triangle to remember these:



Example

A train travels at an average speed of 90 miles per hour. How long does it take to travel 315 miles?

Solution

We need to calculate the time:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{315}{90} = \underline{3.5 \text{ hours}}$$

Practice Questions

- A car is travelling at an average speed of 55 km per hour. How far will it travel in 4 hours?
- On a sponsored walk, Aimee walks 21 miles in 6 hours. What was her average speed? State the units of the answer.
- A cyclist travels at an average speed of 16 miles per hour. How long does she take to cycle 92 miles?
- A plane is flying at a constant speed of 240 metres per second. What distance does it fly in 60 seconds? Give the answer in km.
- Sound travels at 20 km per minute. What time does it take for sound to travel 5 km?

Reminder

Be careful when interpreting time on your calculator.

For example:

- 0.5 hours means $\frac{1}{2}$ hour, which is 30 minutes
- 2.25 hours means $2\frac{1}{4}$ hours, or 2 hours 15 minutes
- 8.75 hours means $8\frac{3}{4}$ hours, or 8 hours 45 minutes

Practice Questions

- (6) A car travels at 70 miles per hour. How far does it travel in 1 hour 30 minutes?
- (7) A car travels 50 miles at an average speed of 40 miles per hour. How long does this take, in hours and minutes?
- (8) A cyclist travels 66 km in 2 hours 45 minutes. Work out the average speed.
- (9) Gabriel ran 30 km at an average speed of 12 km per hour. How long did he run for, in hours and minutes?

Reminder

Take care when hours and minutes are mixed up.

Example

A jogger is running at an average speed of 9 miles per hour. How far does she run in 40 minutes?

Solution

9 miles per hour means she runs 9 miles in 60 minutes.

Minutes	Miles
60	9
20	3
40	6

Answer: 6 miles

Practice Questions

- (10) A car is driving at an average speed of 50 miles per hour. How far does it travel in 30 minutes?
- (11) A helicopter flies 28 miles in 15 minutes. What is its average speed in miles per hour?
- (12) An orbiting satellite travels 240 km in 30 seconds. What is its average speed in km per hour?
- (13) A military jet is flying at 1,800 km per hour. How far does it travel in 5 minutes?

Reasoning and Problem-Solving

- (14) Here is some working from Jane's exercise book.

Plane flies 390 miles in 1 hour 30 minutes

Average speed = $390 \div 1.30 = \underline{300 \text{ miles per hour}}$

Jane has made a mistake.

Explain the mistake, and correct it.

- (15) Alex cycles 12 miles up a mountain at an average speed of 4 mph. He cycles 16 miles down the other side at an average speed of 32 mph. What was his average speed for the whole journey?

Quick Quiz

- (1) Convert 340 grams to kilograms.
- (2) One of the angles of an isosceles triangle is 100° . What are the other two angles?
- (3) Work out $10^3 - 3^3$, without using a calculator.
- (4) What is the lowest common multiple of 10 and 8?
- (5) Solve $\frac{y+4}{2} = 5$.

Review of Session 16

- (1) Jimmy to get to Sheffield in 2 hours. Sheffield is 136 miles away. At what average speed must he drive?
- (2) The average speed of a train is 80 miles per hour. How long will it take to travel 380 miles?
- (3) A marathon runner is running at an average speed of 10.5 km per hour. How far will he run in 80 minutes?

The focus for today's session is...

Working out fractions and percentages

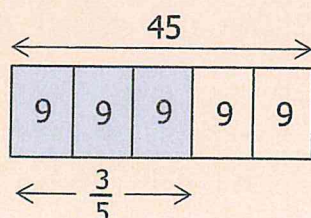
Reminder

To work out $\frac{3}{5}$ of £45:

$$\frac{1}{5} \text{ of } 45 = 45 \div 5 = 9$$

$$\frac{3}{5} \text{ of } 45 = 9 \times 3 = 27$$

This diagram may help you understand the method.



Practice Questions



(1) Work out these:

(a) $\frac{3}{4}$ of 28 (b) $\frac{1}{6}$ of 42

(c) $\frac{7}{8}$ of 40 (d) $\frac{2}{9}$ of 54

(e) $\frac{5}{12}$ of 48 (f) $\frac{11}{30}$ of 180

Reminder

Working out percentages without a calculator

To work out 30% of £52:

$$10\% = 52 \div 10 = £5.20$$

$$3 \times 5.20 = £15.60$$

To work out 4% of £670:

$$1\% = 670 \div 100 = £6.70$$

$$4 \times 6.70 = £26.80$$

$$10\% = \frac{1}{10}$$

$$1\% = \frac{1}{100}$$

Practice Questions



(2) Work out these:

(a) 20% of £640

(b) 70% of 90 kg

(c) 5% of 28 metres

(d) 3% of £420

(e) 15% of 66 cm

(f) 45% of £8

Reminder

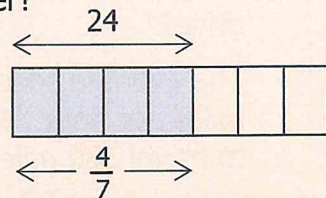
Diagrams can be helpful in solving more difficult problems with fractions.

Example

$\frac{4}{7}$ of a number is 24.

What is the number?

Solution



$\frac{1}{7}$ of the number must be $24 \div 4 = 6$

So the number must be $6 \times 7 = \underline{42}$.

Practice Questions



- (3) $\frac{3}{4}$ of a number is 36.
What is the number?
- (4) $\frac{2}{9}$ of a number is 16.
What is the number?
- (5) Darren has completed $\frac{5}{8}$ of a race. So far he has run 2000 metres. How long is the race?
- (6) I have read $\frac{7}{12}$ of my book.
There are 175 pages left.
How many pages have I read?

Reminder

Here are two ways to work out

63% of £475

using a calculator.

Method 1

Work out 1%, then multiply by 63.

$$1\% = 475 \div 100 = \text{£}4.75$$

$$63\% = 4.75 \times 63 = \underline{\text{£}299.25}$$

Method 2

63% is the same as 0.63.

$$0.63 \times 475 = \underline{\text{£}299.25}$$

Practice Questions

- (7) Work out these:
- (a) 27% of £519
- (b) 8% of 780 metres
- (c) 78% of £5,079
- (d) 52% of £8.50
- (e) 16.5% of 402 kg
- (8) Penny earns £518 per week.
One week, she receives a bonus of 13%. How much is the bonus?
- (9) A theatre has 1,460 seats. 85% of the seats have been sold. How many seats have not been sold?

Reasoning and Problem-Solving

- (10) There are 800 seats on a train.
35% of them are First Class. The rest are Second Class.
All the First Class seats have been booked.
95% of the Second Class seats have been booked.
How many seats on the train have not been booked?
- (11) On my birthday, I received some money. I spent 45% of it on clothes, 21% of it on meals and 23% on a gift for my sister.
£15.40 of the money is left. How much money did I originally receive?

Quick Quiz

- (1) Work out $\left(\frac{1}{2}x\right)^2$ when $x = 8$.
- (2) The perimeter of a regular hexagon is 120 cm. What is the side length of the hexagon?
- (3) What number is half-way between 5.7 and 5.71?
- (4) Which of these is better value for money?
 - a pack of 6 Mars bars costing £4.50
 - a pack of 10 identical Mars bars costing £7.99
- (5) Draw a hexagon which contains exactly two reflex angles, and which has rotation symmetry order 2.

Review of Session 17

- (1) Work these out without a calculator:
 - (a) $\frac{7}{9}$ of 36
 - (b) 15% of £80
- (2) Use a calculator to work out 48% of £705.
- (3) Josh buys some packets of crisps for his son's birthday party.

At the end of the party, $\frac{2}{5}$ of the packets are left.

There are 24 packets left.
How many did Josh buy?

The focus for today's session is...

Sequences

Reminder

Here is a number sequence.

5 8 11 14 17 ...

The numbers in the sequence are called **terms**.

For the sequence above, the rule is:

First term is 5

Add 3 to get the next term.

This is called a 'term-to-term rule'.

In this sequence

3 6 12 24 48 ...

the rule is

First term is 3

Multiply by 2 to get next term.

Practice Questions

- (1) Write down the first five terms of these sequences:
 - (a) First term is 20
Subtract 2 to get next term
 - (b) First term is 2
Multiply by 3 to get next term
- (2) Work out the rules for these.
 - (a) 11 17 23 29 35 ...
 - (b) 80 40 20 10 5 ...
 - (c) 4 20 100 500 2500 ...
- (3) Here is another rule for a sequence.
First two terms are 2 and 5
Add the previous two terms to get the next term.
Work out the next three terms.

Reminder

Another way to describe a sequence is to give an expression for the n th term.

Example

The n th term of a sequence is $4n - 1$.
Work out the first five terms.

Solution

1st term: $n = 1$ gives $4 \times 1 - 1 = 3$

2nd term: $n = 2$ gives $4 \times 2 - 1 = 7$

3rd term: $n = 3$ gives $4 \times 3 - 1 = 11$

4th term: $n = 4$ gives $4 \times 4 - 1 = 15$

5th term: $n = 5$ gives $4 \times 5 - 1 = 19$

First five terms are 3, 7, 11, 15, 19

Practice Questions

- (4) Here are some expressions for the n th term of sequences.
Write down the first five terms for each one

(a) $2n + 5$ (b) $6n - 4$

(c) $100 - 2n$ (d) n^2

- (5) The n th term of a sequence is $10n + 5$. Work out the 15th term.

- (6) The n th term of a sequence is $n^2 + n$. Work out the 20th term.

Reminder

A linear sequence is a sequence which goes up (or down) by the same amount each time.

Here is a way to find the expression for the n th term of a linear sequence.

Example

3 8 13 18 23 ...

Solution

This sequence goes up by 5 each time.
The sequence $5n$ starts like this:

5 10 15 20 25 ...

The given sequence is **2 less** than this,
so the expression is $5n - 2$.

Practice Questions

- (7) Work out expressions for the n th term of each of these linear sequences:

(a) 5 9 13 17 21

(b) 7 17 27 37 47

(c) 13 20 27 34 41

(d) 10 25 40 55 70

- (8) These are a little harder:

(a) 1 3.5 6 8.5 11

(b) 19 18 17 16 15

(c) 55 50 45 40 35

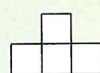
(d) 0.14 0.2 0.26 0.32 0.38

Reasoning and Problem-Solving

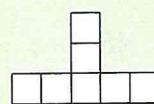
- (9) A linear sequence starts: 4 12 20 28 36 ...
What will be the 1000th term of the sequence?

- (10) Here is a sequence of shapes made from squares.

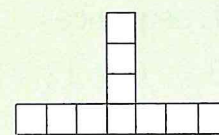
How many squares are there in Shape n ?



Shape 1



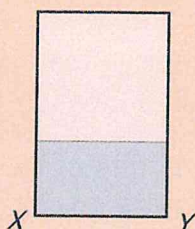
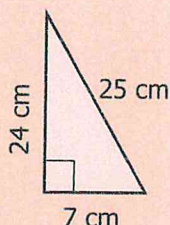
Shape 2



Shape 3

Quick Quiz

- Work out $6 - (-4)$.
- What number satisfies all these:
 - factor of 36
 - multiple of 9
 - greater than 10
- Work out the area of this triangle.
- Work out $\frac{5}{8} - \frac{1}{4}$.
- Which of these best describes the shaded area?



- A points 3 cm from X
- B points less than 3 cm from X
- C points 3 cm from XY
- D points less than 3 cm from XY

Review of Session 18

- What is the next term of this sequence?
62 56 50 44 38 ...
Explain how you worked it out.
- The expression for the n th term of a sequence is $8n + 3$.
What is the 12th term of the sequence?
- Find an expression for the n th term of this linear sequence:
6 10 14 18 22 ...
- Find an expression for the n th term of this linear sequence:
30 25 20 15 10 ...

The focus for today's session is...

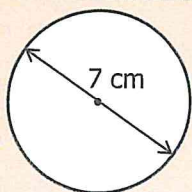
Circumference and area of a circle

Reminder

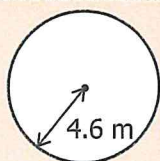
The rule for working out the circumference of a circle is:

$$\text{circumference} = \pi \times \text{diameter}$$

π is the number 3.1416 ... (or use the π button on your calculator).



Diameter = 7 cm
Circumference = $\pi \times 7$
= 22.0 cm (1 d.p.)



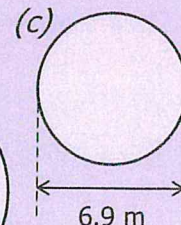
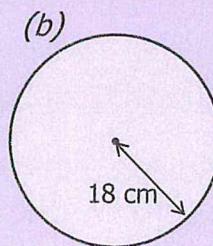
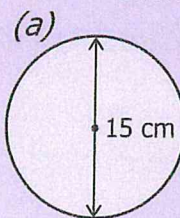
Diameter = $4.6 \times 2 = 9.2$
Circumference = $\pi \times 9.2$
= 28.9 m (1 d.p.)

The rule can also be used in reverse:

$$\text{diameter} = \frac{\text{circumference}}{\pi}$$

Practice Questions

- Work out the circumference of each of these circles.

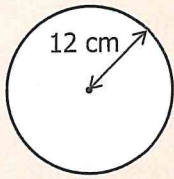


- The circumference of a circle is 20 m. Work out its diameter in m, correct to 1 decimal place.
- The circumference of a circle is 150 cm. Work out its radius, correct to 1 decimal place.

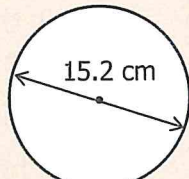
Reminder

The rule for working out the area of a circle is:

$$\text{area} = \pi \times \text{radius}^2$$



$$\begin{aligned}\text{Radius} &= 12 \text{ cm} \\ \text{Area} &= \pi \times 12^2 \\ &= \pi \times 144 \\ &= \underline{452.4 \text{ cm}^2} \text{ (1 d.p.)}\end{aligned}$$



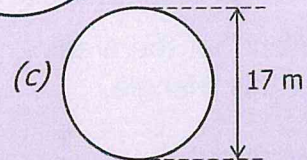
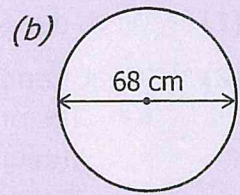
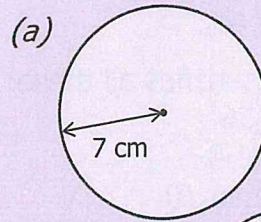
$$\begin{aligned}\text{Radius} &= 15.2 \div 2 = 7.6 \\ \text{Area} &= \pi \times 7.6^2 \\ &= \pi \times 57.76 \\ &= \underline{181.5 \text{ cm}^2} \text{ (1 d.p.)}\end{aligned}$$

To use this rule in reverse:

divide the area by π , then square root

Practice Questions

- (4) Work out the areas of these circles, giving your answers to 3 significant figures.



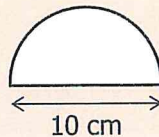
- (5) The area of a circle is 200 m^2 . Calculate its radius in metres, correct to 1 decimal place.

Reminder

If you have a fraction of a circle, work out the complete circle first.

Example

Work out the total perimeter of this semi-circle.



Solution

Circumference of complete circle
 $= \pi \times 10 = 31.416 \dots \text{ cm}$.

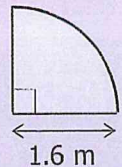
Circumference of semi-circle
 $= 31.416 \div 2 = 15.708 \dots \text{ cm}$

Perimeter $= 15.708 + 10 = \underline{25.7 \text{ cm}}$ (1 d.p.)

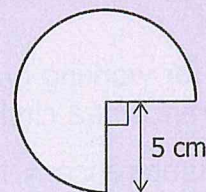
Practice Questions

- (6) Work out the area of the semi-circle in the example.

- (7) Work out the area of this shape.



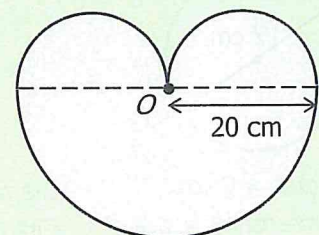
- (8) Calculate the total perimeter of this shape.



Reasoning and Problem-Solving

- (9) The shape on the right consists of three semi-circles. The point O is the centre of the large semi-circle.

- (a) Calculate the total area.
(b) Calculate the total perimeter.

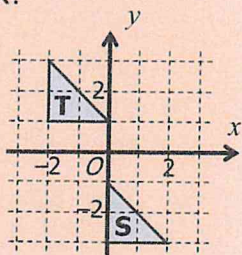


- (10) The diameter of a bicycle wheel is 62.2 cm .

In riding the bicycle 100 metres , how many times does the wheel go round?
Give the answer to the nearest whole number.

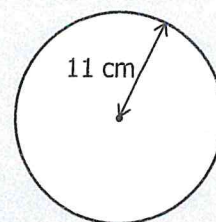
Quick Quiz

- (1) Work out 892×6 , without a calculator.
- (2) Simplify the expression $\frac{x^{20}}{x^4}$.
- (3) Round the number 6.8502 correct to 1 decimal place.
- (4) In a class of 25 students, 11 handed in their homework. What percentage did not hand in their homework?
- (5) Describe fully the transformation that maps **S** to **T**.

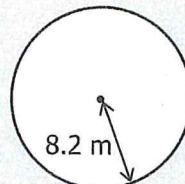


Review of Session 19

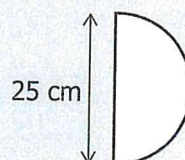
- (1) Calculate the area of this circle.



- (2) Calculate the circumference of this circle.



- (3) Calculate the area of this semi-circle.



- (4) The circumference of a circle is 1 m. Calculate its diameter in cm, correct to 1 decimal place.

The focus for today's session is... Solving ratio problems

Reminder

Example 1

Adam and Brandon share £120 in the ratio 5 : 3. How much does each of them get?

Solution 1

Here is a picture of the ratio 5 : 3.

A					
B					

8 shares altogether

$120 \div 8 = \text{£}15$ in each share

A	15	15	15	15	15
B	15	15	15		

So Adam gets $5 \times 15 = \text{£}75$

Brandon gets $3 \times 15 = \text{£}45$

Practice Questions

- (1) Amanda and Baljit share £150 in the ratio 2 : 3.

How much does Baljit receive?

- (2) In a History exam, there is a short paper and a long paper. The marks for the two papers are in the ratio 3 : 4. The total for both papers is 98.

How many marks are there on the short paper?

- (3) In a school, the ratio of adults : girls : boys is 1 : 5 : 6.

Altogether, there are 372 people in the school. How many of these are children?

Reminder

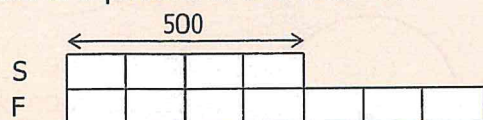
Example 2

For a cake recipe, sugar and flour are mixed in the ratio 4 : 7.

How much flour must be added to 500 g sugar?

Solution 2

Here is a picture of the ratio 4 : 7.



The sugar is 500 g, so each share is $500 \div 4 = 125$ g.

S	125	125	125	125				
F	125	125	125	125	125	125	125	

So amount of flour is $7 \times 125 = \underline{875}$ g

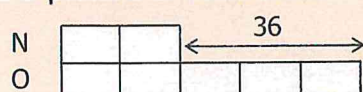
Example 3

Nikesh and Ola each take a History test. Their scores are in the ratio 2 : 5. Ola scores 36 more marks than Nikesh.

How many marks does Nikesh score?

Solution 3

Here is a picture of ratio 2 : 5.



The difference in scores is 36, so each share = $36 \div 3 = 12$.

N	12	12			
O	12	12	12	12	12

So Nikesh scores $2 \times 12 = \underline{24}$.

Practice Questions

- (4) In January, the cost of electricity and gas was in the ratio 3 : 5.

The cost of gas was £45. What was the cost of the electricity?

- (5) In a primary school, the ratio of boys : girls is 9 : 7. There are 30 more boys than girls.

How many pupils are there in the school altogether?

- (6) In a recipe, flour and sugar are mixed in the ratio 3 : 2.

How much sugar is needed if 225 g of flour is used?

- (7) Janine and Kathy win a prize of £480. They share the winnings in the ratio 7 : 3.

How much more does Janine receive than Kathy?

- (8) At school, students receive Merits for good work, and Demerits for bad work.

Stew receives Merits and Demerits in the ratio 6 : 1. He receives 35 more Merits than Demerits.

How many Merits did he receive?

- (9) Tony goes shopping. He spends money on food and clothes in the ratio 4 : 9.

He spends £117 on clothes. How much does he spend altogether?

Reasoning and Problem-Solving

- (10) A car dealer sells three makes of car: Honda, Toyota and Ford.

The numbers of Hondas and Toyotas in stock are in the ratio 8 : 5.

The numbers of Hondas and Fords are in the ratio 4 : 3.

The dealer currently has 12 more Hondas than Toyotas.

How many Fords are currently in stock?